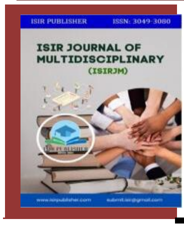


ISIR Journal of Multidisciplinary (ISIRJM)
ISSN: **3049-3080** (Online)
Frequency: Bimonthly
Published By ISIR Publisher
Journal Homepage Link- https://isirpublisher.com/isirjm/

SOLVING ALGEBRAIC PROBLEMS APPLYING COMMERCIAL FACTORIZATION: A QUALITATIVE RESEARCH EXPERIENCE IN SECONDARY EDUCATION

By

Javier Darío Vergara Castiblanco Mg
Faculty of Education. Master's Degree in Didactics of the Mathematics in Secondary and High School Education. International University of the Rioja, The Rioja, Spain

Received: 02/06/2026
Accepted: 08/06/2026
Published: 11/06/2026

Vol – 2 Issue – 3

PP: -53-72

Abstract

The problem-based learning for working on commercial factorization with 4th year secondary compulsory school students allows the application of practical knowledge in the mathematics classroom at an educational center in Bogotá, Colombia (South America).

Algebraic factor decomposition has been developed to describe the commercial management involved in the consumption of products, generating the process of buying and selling goods and services; in order to attract new concepts and experiences of useful, practical, creative, updated, competent knowledge and reflective challenges that are created among school work teams.

The core of this article lies in designing a didactic intervention to improve problem-solving through commercial factorization by applying technology and the collaborative learning wearing the basic economic equations and marketing measures to critical thinking training and assessment processes and analytical thinking in real cases of school learning.

Therefore, the theoretical framework explains the three categories based on the characteristics ranging from arithmetic factorization, algebraic, geometric and commercial with the problem-based learning and the manipulative-digital resources.

The methodology is based on the pedagogical techniques of the active-constructive learning, problem-based learning, commercial learning approach, collaborative learning and game-based learning of meaningful experiences, using

10 cases of algebraic decomposition, applying commercial and virtual learning with Geogebra and Python-Matplotlib which is synthesized in the games-contests.

Diagnostic, formative and summative assessments, as described in the rubric, were also carried out using the MAXQDA 2020 software program with qualitative data on cognitive tasks. The participants were eight heterogeneous and normal students of both sexes (four boys and four girls) without learning difficulties, grouped into four pairs for 24 work sessions in the library and computer lab.

The results indicate that students will solve applied problems using the heuristic method, generating knowledges and applying the analysis and use of graphs with business data. The effectiveness of the algebraic model, based on solving commercial factorization problems, is demonstrated, strengthening consume culture.

Keywords: *Heuristic Method, Collaborative Learning, Algebraic Factorization, Problem-based Learning, Didactic Intervention, Compulsory Secondary Education, Commercial Management.*

Abbreviations: *STEM, AAC, PBL, BLA, CL, GBL, COPISI, LCM, GCD.*

1. INTRODUCTION

This work is structured around STEM education (science, technology, engineering and mathematics) with constructivist approach, without to parcel knowledge and based on the tinkering (learning by doing) which includes problem-solving or challenges of type commercial factorization and developing technological applications; being a novel, real, original, innovative, current, unpublished, applied and creative didactic proposal that describes the ten cases of algebraic decomposition with basic economic equations, marketing magnitudes and collaborative learning that seeks to train and

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

evaluate in the meaningful learning.

The algebraic factorization has been studied in an isolated and fragmented way, with the learning of the algorithm predominating in a mechanical-routine manner, without the apprehension of concepts in the students.

The construction of variables, equality relations and equations connects algebraic thinking with geometric sense through signs, words, symbols, processes, and figures of abstract ideas from semiotic language, generating the factorization (Rivera, 2020).

The problem-based learning model is established, which fosters individual group creativity and curiosity to promote critical thinking and analytic with new creative ideas in the construction of knowledge of divergent thinking, open-mindedness, and reflective debates that range from the theoretical to the practical (Zhou & Navarro, 2025).

In secondary students, myths and phobias are created with some cognitive obstacles to understanding, operational mastery, dynamization and development of learning, generating some apathy, cognitive distrust, anxiety, insecurity, tension, traumas, stress, boredom, fear and frustration during their teaching and learning of meanings. The words expressed by students, such as the “coconut of mathematics”, “math karma” or “mateburros”, indicate the different cognitives difficulties of learning and of applicability in real environments where the didactic resources will be used, ranging from the concrete, pictorial and abstract (Fernández et al., 2020).

The algebraic operationalization describes the generalities of quantities expressed in unknowns that combine the mathematical objects and its mastery requires much analytical practice, knowing the basic and derived forms to know how to decompose any expression, always arriving at the same result (García & Puyol, 2024).

Furthermore, they will be able to create and recreate the meaningful learning, based on the commercial concept for business profitability in cognitive tasks related to consumption of products, consolidating deductive reasoning, literacy, group work, and a vocational sense of realities applied to structural algebra (Loor, 2022).

Also, the metacognitive process will stimulate learning, not by being passive recipients of information, but by fostering active attitudes of motivation, social integration, critical thinking, autonomy and personal initiative; enabling students to seek precise and coherent solutions in group work, improving self-learning, significance of content and avoiding the conflicts in the classroom (Vargas, 2021).

2. PROBLEM STATEMENT

This article is based on difficulties that of 4° secondary school students have with respect to the study of algebra, indicating skeptical attitudes of demotivation, rejection and cognitive ignorance of applications to daily life, affecting their academic performance.

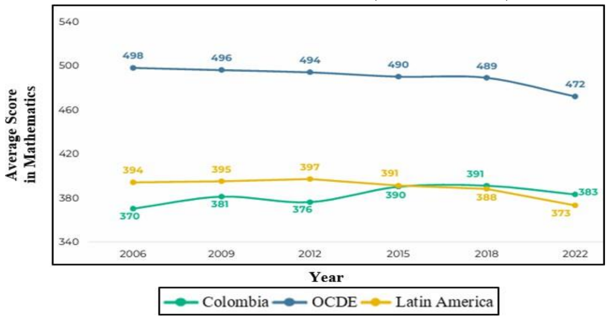
With this type of education practice will prepare them for the future jobs by developing mathematical and digital skills with project-based strategies focused on self-esteem, cultural exchange, proactive and collaborative learning; promoting leadership, creative attitude and linguistic,critic and reflective development among classmates (Ramos & Núñez, 2024).

The challenge of 21st-century education is based on being, knowing, learning to think and knowing how to do, where pedagogical knowledge will be learned collaboratively, based on observation, development of mental processes, problem-solving and games-contests with constructive learning.

Colombia's test results showed a score of 383 out of 500 with 7804 students. This is compared to the OCDE average of 472 points and the Latin American average of 373 points, as shown in figure 1 which includes the overall mathematics scores for Colombia, the OCDE and Latin America between 2006 and 2022.

Colombia falls below the average of the 81 OCDE countries. According to the 2022 PISA mathematics tests, Colombia ranks 64th globally and is among the top six performers in Latin America (ICFES, 2024).

Figure 1. Overall score in mathematics between Colombia, OECD and Latin America (2006 to 2022).



Source: Adapted from the PISA 2022 Report (OECD 2023).

The following research question is described: Why is problem solving through algebraic factorization applicable to commercial management and technological virtual environments?

Working Hypothesis

As possible solutions to the problem include strengthening symbolic-algebraic language, expanding critical and analytic thinking to solve commercial mathematical actions,

using the heuristic method and improving collaborative group coexistence, applying the communication, concentration, mental agility and the cognitives habilities.

3. LITERATURE REVIEW

Arithmetic Factorization Characteristics

Prime or arithmetic factorization is a process that begins in primary school, is reinforced in secondary school, and is deepened with practical applications in high school.

The factors of a number are equal to its divisors, where prime or first numbers are those that can only be divided by

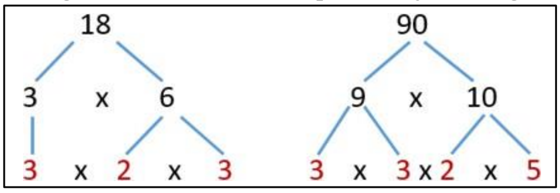
*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

themselves and one, such as {2, 3, 5, 7, 11, 13, 17, 19, 23, 29 …}, being infinite numbers and are located in the Sieve of Eratosthenes as a practical and ancient method (Bernaschini, 2017).

There are two algorithmic methods for finding the prime factors of a number using the factor tree diagram and performing successive divisions. The development of arithmetic algorithms generates knowledge with finite and defined characteristics of inductive reasoning, using flowcharts with graphical representations and pseudocodes of qualitative sequences (Castellano & Sandoval, 2022).

The first decomposition method consists of a pair of factors of the natural number or positive integer where the sequential chain is carried out, simplifying them to the maximum until the result is found which will be the prime numbers where the figures increase, as represented in the figure 2 which describes prime factorization.

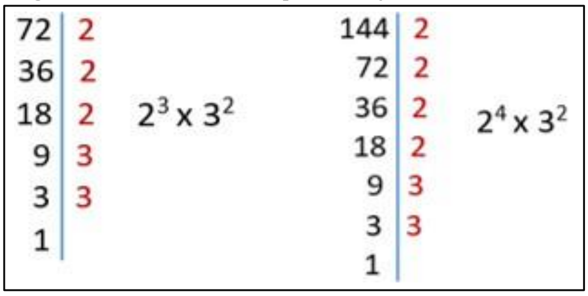
Figure 2. Arithmetic decomposition by tree diagram.



Source: Own elaboration.

The second method of decomposition consists of finding the prime factors of anatural number, making continuous divisions (one after another) between the smallest divisible prime number, operating and writing the quotient below the resulting number, simplifying it in order (half, third, fourth, fifth, etc.) until obtaining 1 as its smallest expression of the quotient, drawing a vertical line to the right of the positive integer, represented in figure 3 that describes the prime factorization.

Figure 3. Arithmetic decomposition by successive divisions.



Source: Own elaboration.

Furthermore, the fundamental theorem of arithmetic is analyzed, noting that every positive integer greater than 1 can be uniquely expressed as the product of prime numbers, as indicated in the prime factorization. That is, every integer other than 1, 0 and -1 can be uniquely decompose; where this concept being like the product of prime numbers, a concept explained by Euclid of Alexandria in the history of mathematics (Trejos, 2019).

Algebraic Factorization Characteristics

Algebraic factorization is a process that begins at the end of secondary education, continues with practical exercises in applied high school and is deepened at university through scientific research.

Its study is based on the decomposition of expressions and properties of addition and subtraction of terms indicated in the product of two or more factors.

It is a powerful and important mathematical tool that simplifies complex polynomials with operational notions, described in the 10 factorable cases of symbolic representations of variables or letters and of constants or numbers with geometric theorems in the divisors or factors and that when multiplied together give the result of the product of the first expression.

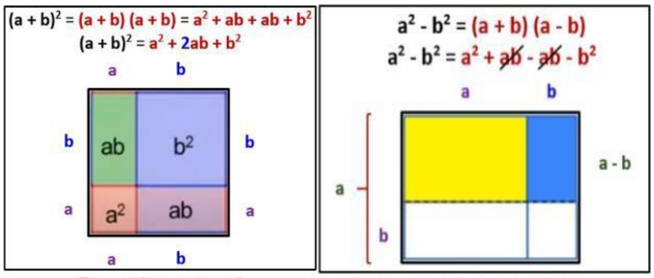
The algebra is based on the arithmetic generalization of numerical quantities, seeking to cognitively develop problem-solving processes through the Euler model, which is based on the representation of the positive or natural integer.

The most effective method for factorization describes the sum of two squares as: $N = a^2 + b^2 = c^2 + d^2$. Besides, Fermat's method includes the representation of an odd natural number as the difference of two perfect squares, $n = c^2 - d^2$ and the operation is factored as $(c + d)(c - d)$, with none of its factors equal to 1 with respect to n (Bolaños & Segovia, 2021).

The algorithmic mental structures of algebraic factorization are evaluated in the abstract application with real numbers, sums, differences, quotients, powers, matrices and polynomials that are described by the multiplication or product operation, obtaining the distributive property.

In this way, the operation is reduced to its simplest form to express it as a product, applying the relationships, properties, grouping symbols, notable products, first and second degree equations that reinforce the resolution of problems with abstract geometric methods for their spatial analysis (Flores et al., 2017).

Also, the fundamental theorem of algebra or D'Alembert's



Binomial sum squared | Difference of two perfect squares

theorem is analyzed in its mathematical structure, indicating that every polynomial of coefficients has two roots of the complex number estimated at 0.

This is demonstrated by developing the operations of successive divisions of polynomials into linear factors where complex numbers are described as a closed field, working with the simple quadratic function with difference of imaginary perfect squares, represented in figure 4 as factorization (Ortiz, 2019).

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Figure 4. Linear decomposition of an imaginary quadratic function.

$f(x) = x^2 + 25$	$\sqrt{-25} = x$
$0 = x^2 + 25$	$\pm 5i = x$
$-25 = x^2$	$f(x) = (x + 5i)(x - 5i)$

Source: Own elaboration.

Geometric Factorization Characteristics

Geometric factorization is an applied process that begins at the end of secondary education, is complemented in high school and is expanded at the university level with professional studies.

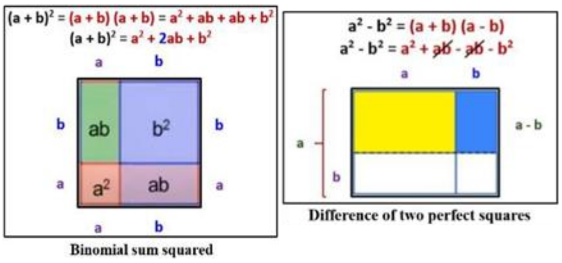
It is based on the development of algebraic decomposition from a geometrized or spatial perspective of critical thinking, promoting the resolution of everyday problems through the properties of plane figures such as triangles, polygons, quadrilaterals and circles. Also, the properties of three-dimensional geometric bodies, such as polyhedra and solids of revolution, are analyzed, evaluating distances, areas, perimeters, line segments, angles, volumes and surfaces.

Algebraic factorization began with the Babylonians, through the discovery of irrational numbers. The Greeks addressed geometric problems based on measurable needs of real objects, requiring generalized algebra, based on ancient arithmetic, to solve applied geometric-spatial problems using quadratic equations and areas.

Using the distributive property of $a \cdot (b + c + d) = ab + ac + ad$, we obtain the rectangle defined by **a** and by the separate sum of segments **b**, **c** and **d**. The identity $(a + b)^2 = a^2 + 2ab + b^2$ is contained within two squares and two identical rectangles whose four different areas are added together, based on the product of two binomials with the same sign, through the distributive property.

The difference of squares $a^2 - b^2 = (a + b)(a - b)$ allows us to find the difference of areas by means of the product of two binomials with opposite signs, developing the distributive property, described in figures 5with geometric interpretations of the sum of squares of a binomial and the difference of perfect squares (Vasco, 2023).

Figures 5. Geometry of square sum of the binomial and difference of two perfect squares.



Source: Own elaboration.

Geometry is a practical tool for critical, logical and thoughtful thinking in algebraic factorization. The aim is to strengthen the natural-symbolic semantic language of signs and syntactic tools with mathematical applications that transform it into graphic-visual representations, creating algorithms and structures with sequential properties of geometric objects (De León & Ávila, 2024).

Commercial Factorization Characteristics

Commercial factorization constitutes a process of mathematical extension, based on arithmetic or prime, algebraic and geometric decomposition, where the new concept applied to business management is described, whose foundation is commerce from the marketing and current economics of national and international businesses, based on culture of products consumption (goods and services).

The company in its origins, emerged in the middle ages during feudalism and was composed of small artisans organized into guilds who sold their handcrafted products for trade. Consequently, the Phoenicians were considered the first merchants in the capitalist market system of the society. Competitive companies follow a process of trial and error for their business growth; because the difference between companies is not in the product or service, but in the added value offered to customers to be satisfied consumers of their goods, offering reliability and quality of consumption. Likewise, every company must generate wealth in capitalist societies and the factors of production are combined, based on land, work, capital and entrepreneurship, to achieve large-scale economic development (Tantalean, 2022).

Purchase motivations are based on factors such as economics, quality, personal satisfaction, prestige, security, and social recognition. It is understood that goods not only serve to provide food, protection, or clothing; they also create a sense of belonging, reflecting ways of thinking, being, feeling, and acting, focused on social behaviors described in beliefs, attitudes, tastes, preferences and desires according the customer emotional (Muñoz et al., 2019).

Marketing is a practical and creative process that involves commercial and social thinking. Furthermore, it is a mercantilist concept related to the exchange of products such as goods, food, tools, clothing, and services that benefit others, based on barter, the oldest form of commerce, without the use of money.

It is founded on buying and selling, based on demand (families) and supply (products) with a market equilibrium point. The marketing mix or four Ps is the commercial process for making the sales, based on prices, products, promotions and distribution or place, aimed at satisfied customers who choose purchases with psychological perspectives of perception, motivation, learning, emotion and social influence for brand positioning as indicated in table 1 with productive characteristics (Rodríguez et al., 2020).

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Table 1. Marketing mix analysis 4A.

ACCESSIBILITY	Customers who use the products and services with availability and convenience.
AFFORDABILITY	Customer willingness to pay a price for the product or service.
AWARENESS	Product and brand knowledge.
ACCEPTABILITY	It satisfies the needs of customers for products or services.

Source: Adapted from the scientific article - Modern Marketing Trends, a Theoretical Review, 2020.

Problem Based Learning

Problem-based learning (PBL) is based on STEM education from a constructivist, dynamic, autonomous, critical and mutual supportive approach that connects the school with the real world, participating in collaborative learning in which participants interact with each other, taking into account inclusion in diversity.

Students will work in small groups to develop constructive learning activities in problem solving in a coordinated manner, with the guidance of the teacher-guide, generating cognitive debates of theoretical-practical knowledge. This includes the division of tasks among the group members where each one will contribute cognitive ideas for the expository debates, generating motivation, social skills, improved learning, critical thinking, and preparation for the working life (Damián et al., 2021).

Mathematical problem-solving is evaluated with a cross-cutting perspective in other areas of knowledge with reflective reasoning skills, so that the student makes decisions in a creative and innovative way with interpretive approaches to symbols and representations, argumentative approaches to statements-actions and propositional approaches to activities with brainstorming (Patiño et al., 2021).

Therefore, the resolution of algebraic, arithmetic, and geometric problems is based on understanding the question, developing the content, planning the action, solving and verifying the logical solution processes, through the heuristic method; without falling into the mechanical routine of superficial and immediate answers (Gasco, 2017).

The mathematical analysis is strengthened by seeking to contrast, analyze alternative and extreme cases, establish objectives, compare, consolidate and new review concepts, solve everyday life processes,

create practical problems for other sciences of knowledge and seek mathematical connections, based on mental schemes of meaningful learning, as described in table 2 which explains the process of the heuristic method (Mendoza et al., 2024).

Table 2. Steps of the heuristic method.

1. Understanding the Problem.	⋆Read the problem. ⋆Identify unknowns and data. ⋆Discover the relationships. ⋆Recognize relevant information.
2. Plan Design.	⋆Organize information. ⋆Plantear el problema de otra forma. ⋆Observe if all the information is used
3. Operational Development.	⋆Solve each planned step. ⋆Reflect on its usefulness.
4. Process Review.	⋆Check what was requested. ⋆Find out if there is another solution. ⋆To pose new problems. ⋆Support your conclusions with arguments.

Source: Own elaboration.

Also, the strategy of learning to think, learning to learn, and learning to apply is established, which leads to the mental skills that improve social attitudes and analytical thinking; allowing progress in educational research by combining literacy and the technology (Bermúdez, 2021).

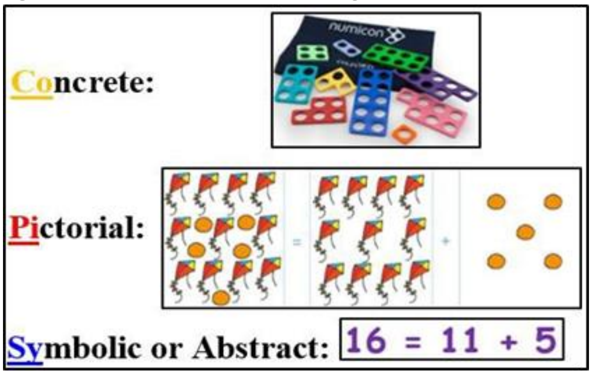
Manipulative and Digital Resources

Concrete manipulative resources are conventional material representations used to learn abstract mathematical meanings, facilitating the teaching learning processes of school content through constructive thinking that facilitates problem-solving and technological development with new cognitive advances. In addition, digital resources are informative with electronic material that can be consulted on computers via the internet.

Specifically, the manipulable materials in mathematics are used in the early childhood, primary, secondary and high school stages, applying the Singapore method with project-based learning that describes constructive, dynamic, analytical, critical, reflective, creative, practical and investigative thinking (Gallego, 2024).

The students express joy, confidence, motivation to learn and creative curiosity; taking into account the processes of knowledge, from the *concrete* with practical support material, the *pictorial* with visual information from drawings and the *symbolic or abstract* with numerical representations in mental processes, as seen in Figure 6, which describes the COPISI didactic model or Singapore method (Jiménez & Espinosa, 2019).

Figure 6. Use of the COPISI teaching method of a number.



Source: Own elaboration.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Manipulation of structured and unstructured material didactics in mathematics is established, based on form, color, dimension and texture, which leads to generating strategies to create and recreate applied learning from the basic to complex concepts (González, 2022).

This manipulable and virtual teaching resources generate a fun, enjoyable and useful study experience for students, based on mental agility, concentration, memory, observation and group interaction (Zambrano et al., 2025).

The obstacles to using teaching resources are evaluated, particularly the fact that students often end up playing with the materials instead of using them for learning. There is also the difficulty in training students in their use. Manual construction demands pedagogical instructional time. Purchasing these materials is costly for the teacher, who incurs expenses without financial support from the school. Excessive use of teaching materials will be avoided to prevent falling into a routine.

The use of dangerous materials that threaten students' health or encourage harmful habits and violent vices is prohibited. Frequent use of electronic-technology devices can lead to psychological addiction and physical health problems (Pozo, 2020).

4. METHODOLOGY

Type of Study and Focus Techniques

This study adopted a qualitative research design; being appropriate for analyzing mathematical data in arithmetic, algebraic, geometric and commercial factorization where the variables are not manipulated.

The methodological learning techniques used in the didactic proposal in mathematics education, are: It is based on *problem based learning* (PBL), applying the heuristic method in 10 cases of study in commercial factorization, combining the economics and business marketing. Arithmetic factorization will be applied to prime factorization problems. Algebraic factorization will be applied to the geometry, administration, architecture, medicine and agriculture.

The process includes *business learning approach* (BLA), which develops consumer skills in commercial management using economic equations related to prices, revenue, profits, discounts, sales, and net worth. Marketing units are used, based on measures of price, percentages, lengths, areas, volumes, heights, maximum and minimum production quantities (batches), packaging types, weights, sales forecasts and products quantities.

The 10 cases of algebraic factorization (Baldor, 2016) with practical development in commercial management:

a) ***Case 1: Common Factor*** (*Pre-sale price of product in pants and shirts stores*).
b) ***Case 2: Common Factor by Grouping Terms*** (*Pre-sale prices for two men's and women's footwear products*).
c) ***Case 3: Perfect Square Trinomial*** (*Quantity of defective product in canned sweets of the fruit cocktail*).
d) ***Case 4: Difference of Perfect Squares*** (*Construction of a winery at production company of food*).
e) ***Case 5: Perfect Square Trinomial by Addition and Subtraction*** (*Amount of sweetener in a gaseous product*).
f) **Case 6: Trinomial of the Form x2 – bx + *c*** (*Minimum and maximum production quantity of an edible product)*.
g) **Case 7: Trinomial of the Form ax^2 – bx + *c*** **(***Number of packages of the packaging in the distribution of products in courier logistics services*).
h) ***Case 8: Perfect Cube of Binomials*** (*Nutritional calorie quantity and its net commercial value of the chocolate dietary product*).
i) ***Case 9: Difference of Perfect Cubes*** (*Number of empty and occupied rooms in hotel with total production percentages*).
j) ***Case 10: Difference between two Equal Powers*** (*Quantity sold of high and medium range bicycles with forecast of future sales in franchise*).

- This pedagogical model is based on *active constructive learning* (AAC), developing a dynamic content process with meaningful practical experiences, based on critical-analytical thinking to explore knowledge in problem-solving applied to the real environment of factorization.
- *Collaborative learning* (CL) is described, based on the interaction of the target group who share mathematical thoughts in pairs and commit achieving new knowledge, applying brainstorming with constructive debates and evaluating theory with practice through coexistence.
- *Game based learning* (GBL) is implemented in the classroom, describing the didactic activities with virtual and analog educational resources that complement mathematical knowledge.

This approach combines collaborative work, which fosters excitement, joy, motivation, and fun, with creative skills that spark interest, participation and the overcoming of cognitive challenges. The physical materials include games such as Math Wheel of Fortune, Math Pairs, Close the Box, Math Bingo, Algebraic Dominoes and Who Wants to Be a Math Star? The digital resource is the interactive online quiz (Quizizz), which is based on dynamic competitive video games.

Participants and Sample

This is a national government public school located in southeastern Bogotá, Colombia (South America). The school operates on a morning schedule, following Calendar School A, which runs from January to November of the academic year.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Most participants live in the neighborhoods surrounding the school, although a small number reside in other parts of the city, totaling 8 students between 15 and 16 years old (four boys and four girls) with diverse normal cognitive abilities and attention disorders are contextualized within a lower-middle socioeconomic stratum.

It is considered an exploratory, intentional or convenience sample in which students of both sexes are enrolled in the fourth year of compulsory secondary education and were randomly selected from the school database and with a voluntary decision, based on their academic performance both low and high, which has been individually assessed with respect to the judgment decisions of the mathematics teacher and being approved by the author-researcher of the project. For collaborative work, students were divided into four paired groups.

Information Instruments

Data collection involved a semi-open, structured, evaluative "pre" survey based on a script of 14 questions or categories and 76 subcategories of learning with pedagogical-mathematical characteristics in variables such as factorization, reading and writing, problem-based learning, heuristic method, difficulties in solving critical and analytical thinking exercises, computer use and future professional projection.

Information was also collected through the Likert evaluative attitude “post” scale with ordinal variables (Agree, Neutral and Disagree) and is based on a script of 21 items or indicators with cognitive, affective and conative development characteristics, regarding the resolution of problems or challenges in commercial factorization and structured in application processes through Python-Matplotlib and Geogebra.

Therefore, the information is analyzed through the contrast between the 4 types of factorization, taking into account the structure of codes created in the MAXQDA 2020 computer program that describe the coded segments, comparing the statistical data and evaluating the thematic context with criterion of variables for a total of 688 codes, where the results of arithmetic factorization are of 72 codes, algebraic-geometric factorization with 172 codes and commercial factorization with 444 codes applied at the secondary educational level.

Finally, Likert attitudinal scale structure evaluated in the students, considering attitudes (The Cognitive, Affective and Conative), described by items and gender in pie chart, as are the Questions 1, 4, 7, 10, 13, 16, and 19 (Cognitive); Questions 2, 5, 8, 11, 14, 17, and 20 (Affective); Questions3, 6, 9, 12, 15, 18 and 21 (Conative). Female(initial 4 students: 1, 2, 3, 4) and male (fina l4 students: 5, 6, 7, 8) genders are analyzed in the overall histograms respective for the project decisions.

Application of Procedures

Initiation phase: It consists of informed consent from parents of family and students that have been authorized for the development of fieldwork. Also, includes selecting the sample size of participants, who are chosen by the responsible mathematics teacher and approved by the project researcher.

Development Phase: It includes training of selected students and the application in problem solving through the use of virtual programs developed in Geogebra and Python-Matplotlib.

Closure Phase: Evaluate the analyses of the contrasted fieldwork and the application of instruments (structured survey, diagnostic test arithmetic factorization, diagnostic test algebraic and geometric factorization, summative test commercial factorization, Likert attitude scale, apply co-evaluation and self-evaluation).

Applied Data Analysis

The information units are described, which allow flexible coding with abbreviated alphanumeric values for the analysis of thematic content, grouped by categories or indicators according to the script of the questions, applying the EXCEL 2016 computer program and the modern MAXQDA 2020 software that allows graphing the different characteristics of each instrument to diagnose the school improvement plan in applied mathematics.

The groups of 4th year secondary [4°S] school students are represented with capital letters and their assigned digit, pointing the numerical position by the researcher to each collaborative team. Likewise, the number of students is denoted as [A1], [A2], [A3], [A4], [A5], [A6], [A7], [A8]. The question [P] and references [R] are also described, including the corresponding item numbers, as shown in Tables 3, which indicates the structured encoding.

Tables3. Structured coding in different applied instruments.

Semi-Open Structured Survey "Pre"

CATEGORIES	STUDENTS	REFERENCES
Synonym for factorization.	[A1] [A2] [A3] [A4] [A5] [A6] [A7] [A8]	[R1] - [R5]
Applicable factorization.		[R6] - [R7]
Learning obstacles.		[R8] - [R13]
Ability for solving applied exercises.		[R14] - [R23]
Difficulty for solving applied exercises.		[R24] - [R33]
Appropriate academic with expectations performance.		[R34] - [R35]
Difficulties in mathematical literacy.		[R36] - [R41]
Problem statement mathematics.		[R42] - [R45]
Uses of the computer.		[R46] - [R50]
Analog manipulative teaching resources or playful games.		[R51] - [R52]
Computer applications in the mathematics.		[R53] - [R54]
Difficulties in the factorization exercises.		[R55] - [R63]
Careers paths for the future.		[R64] - [R72]
Understandable and up-to-date semi-open structured survey.		[R73] - [R76]

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Likert Attitudinal Scale "Post"

INDICATORS	STUDENTS	ITEMS
Knowledge in arithmetic, geometry, algebra, economics and marketing.	[A1] [A2] [A3] [A4] [A5] [A6] [A7] [A8]	[P1]
Motivation for learning in algebraic factorization.		[P2]
Enjoyment of solving factorization processes.		[P3]
Recognition of trade in factorization problems.		[P4]
Motivation for applications of factorization in other areas of knowledge.		[P5]
I like the algebraic factorization class.		[P6]
Update on problems applied in commercial factorization.		[P7]
Images motivation in commercial factorization problems.		[P8]
Enjoyment of math classes taught by school teachers.		[P9]
Recognition of the heuristic method in solving business problems.		[P10]
Motivation in writing problem statements in commercial factorization.		[P11]
A preference for remaining silent, without clarifying doubts in factorization classes.		[P12]
Recognition of the application of Geogebra in systems classes.		[P13]
Motivation for collaborative work in pairs.		[P14]
Enjoyment of learning problems applied to commercial and geometric factorization.		[P15]
Creation and recreation of manipulative resources with games and contests in factorization.		[P16]
Motivation to implement business opportunities with commercial factorization.		[P17]
Interest in social entrepreneurship classes.		[P18]
Recognition of Python-Matplotlib programming in systems classes.		[P19]
Motivation for problem-based learning to apply the STEM model.		[P20]
Likert attitudinal scale content and design preferences.		[P21]

Sources: Own elaborations.

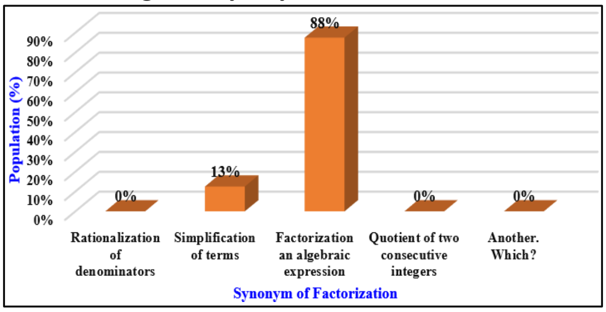
The results of each instrument will be analyzed, evaluating and making decisions in the form of a debate to generate a learning improvement plan in applied factorization.

5. RESULTS AND DISCUSSION

Semi-Open Structured Survey "Pre"

***Question 1:** "Criteria regarding the synonym for factorization"*, describes the primary answer choice [R3]-c, based on the factorization of an algebraic expression with maximum value of 88% and the secondary answer choice [R2]-b, which indicates the simplification of terms with minimum value of 13%. Conceptual decision is correct and all four men marked correct option [R3]-c, according figure 7.

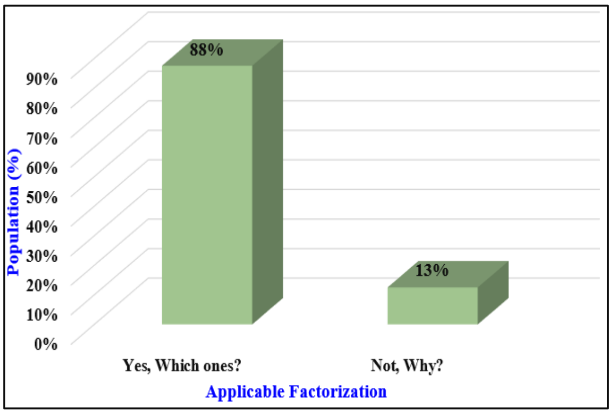
Figure 7. Synonym for factorization.



Source: Own elaboration.

***Question 2:** "Factorization applicable in other fields of real life"*, describes the primary answer choice [R6]-a, based on yes, with 88% maximum value and the secondary answer choice [R7]-b, indicated the not, with 13% minimum value. Answer 1, is resource organization, engineering, programming, costs, economics and business processes. Answer 2, is a lack of knowledge for applying factorization to everyday life. Conceptual decision is all four men marked the option [R6]-a, according figure 8.

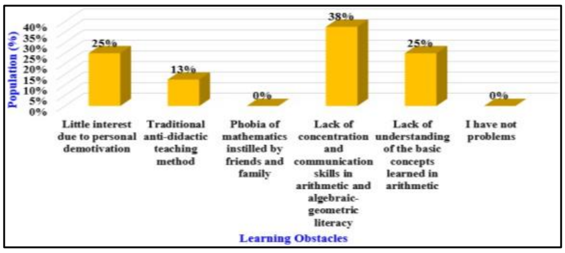
Figure 8. Applicable factorization.



Source: Own elaboration.

***Question 3:** "Obstacles to learning in factorization"*, describes the main answer choice [R11]-d, based on a lack of concentration and communication skills in arithmetic and algebraic-geometric literacy with a maximum value of 38% and the secondary answer choice [R9]-b, which indicates the traditional, anti-didactic teaching method with a minimum value of 13%. Conceptual decision is that 3 women marked option [R11]-d, and 1 woman marks option [R8]-a, generating a greater preference for the first response option, according to figure 9.

Figure 9. Applicable factorization.

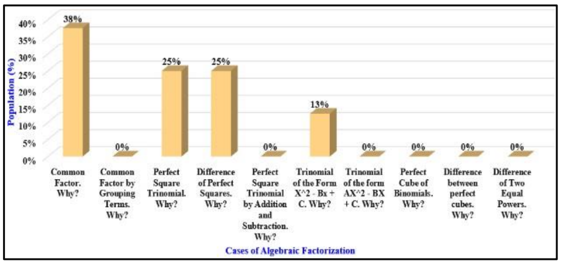


Source: Own elaboration.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Question 4: "Ease in solving application exercises", describes the primary answer choice [R14]-a, based on the common factor with maximum value of 38% and the secondary answer choice [R19]-f, indicated by the trinomial of the form X^2 - Bx + C with minimum value of 13%. Answer 1, indicates that the first factorable case is the simplest and easiest to recognize. Answer 2, indicates that the algebraic structure is easily identified for prime factorization. Conceptual decision is that 2 girls selected option [R14]-a, and 2 girls selected options [R16]-c and [R17]-d, according figure 10.

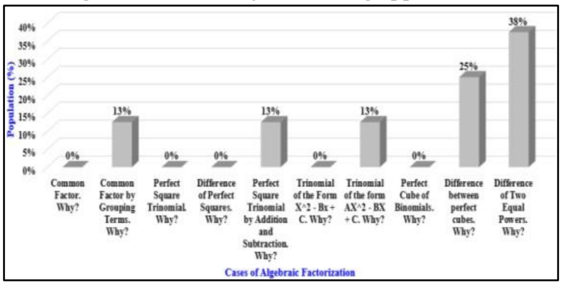
Figure 10. Ability for solving applied exercises.



Source: Own elaboration.

Question 5: "Difficulty solving application exercises", describes the main answer choice [R33]-j, based on the difference of two equal powers with maximum value of 38% and secondary answer choices [R25]-b, indicated in the common factor by grouping of terms, [R28]-e, based on the perfect square trinomial by addition and subtraction and [R30]-g, structured in the trinomial of form AX^2 - Bx + C and all with minimum value of 13%. Answer 1, indicates difficulty with factoring large powers and polynomial division. Answer 2, identifies the obstacle to applying the concept to real-life situations. Answers 3 and 4, assess the high complexity of the process in their understanding. Conceptual decision is that 2 women marked option [R33]-j, and 2 men mark options [R28]-e, [R32]-i, according figure 11.

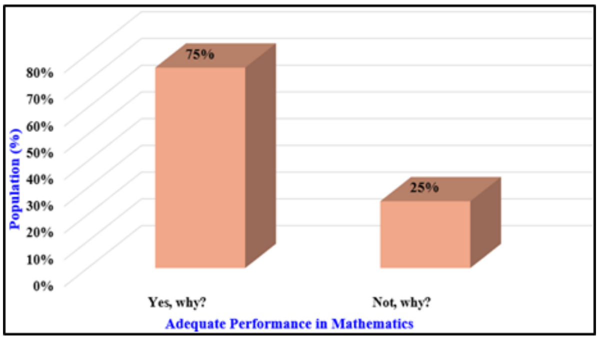
Figure 11. Difficulty for solving applied exercises.



Source: Own elaboration.

Question 6: "Academic performance mathematics that adequate with cognitive learning expectations", describes the primary answer option [R34]-a, based on yes, with maximum value of 75% and the secondary response option [R35]-b, indicated the not, with minimum value of 25%. Answer 1, represents improvement in academic performance, learning of new topics and motivation. Answer 2, represents difficulty understanding operations and a lack of personal effort learn. Conceptual decision is that all four women selected option [R34]-a, according figure 12.

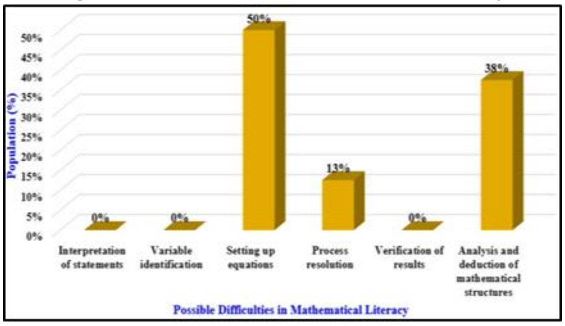
Figure 12. Appropriate academic with expectations performance.



Source: Own elaboration.

Question 7: "Possible difficulties in mathematical literacy", describes the main answer choice [R38]-c, based on solving equations with maximum value of 50% and the secondary answer choice [R39]-d, indicated in process resolution with minimum value of 13%. Conceptual decision is that 2 women marked option [R38]-c, and 2 men marked option [R41]-f, according figure 13.

Figure 13. Difficulties in mathematical literacy.

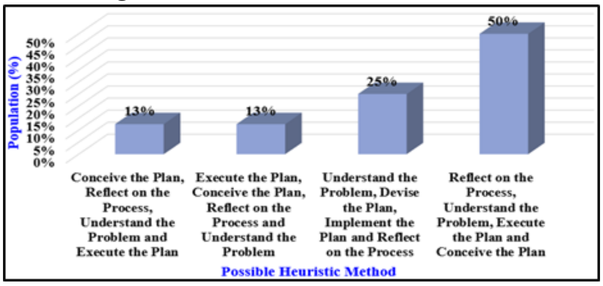


Source: Own elaboration.

Question 8: "Correct way to pose a problem in mathematics with applied ***learning solution***", describes the main answer choice [R45]-d, based on the criteria of reflecting on the process, understanding the problem, executing the plan and conceiving the plan, with maximum value of 50% and the secondary answer choices [R42]-a, including the criteria of conceiving the plan, reflecting on the process, understanding the problem and executing the plan. In addition, [R43]-b, is based on criteria of executing the plan, conceiving the plan, reflecting on the process and understanding the problem. These have minimum value of 13%. The conceptual decision is that 2 women and 2 men marked option [R45]-d, 1 woman marks option [R42]-a, and 1 man marks option [R43]-b, 1 woman marks option [R44]-c, and 1 man marks option [R44]-c. The correct answer is criterion [R44]-c, according figure 14.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

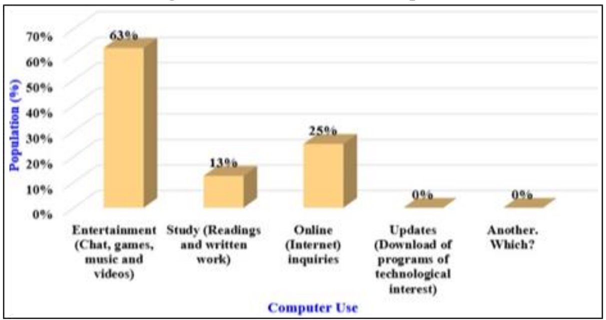
Figure 14. Problem statement mathematics.



Source: Own elaboration.

Question 9: *"**Uses of computer**"*, describes the main answer option [R46]-a, based on leisure (Chat, games, music and videos) with maximum value of 63% and the secondary response option [R47]-b, indicated in the study (Readings and written work) with minimum value of 13%. Conceptual decision is that 3 women marked option [R46]-a, and 1 woman marks option [R47]-b, according figure 15.

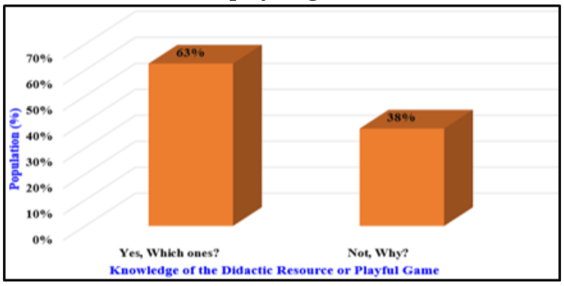
Figure 15. Uses of the computer.



Source: Own elaboration.

Question 10: *"**Manipulative resource or game learning mathematics**"*, describes the primary answer choice [R51]-a, based on yes, with maximum value of 63% and the secondary answer choice [R52]-b, indicated the not, with minimum value of 38%. Answer 1, is internet games, dominoes, Cuisenaire rods, bingo, chess, Sudoku, mazes and lottery. Answer 2, is that these activities are not considered important for learning and do not know how to use them. Conceptual decision is that 3 men selected option [R51]-a, and 1 man selected option [R52]-b, according figure 16.

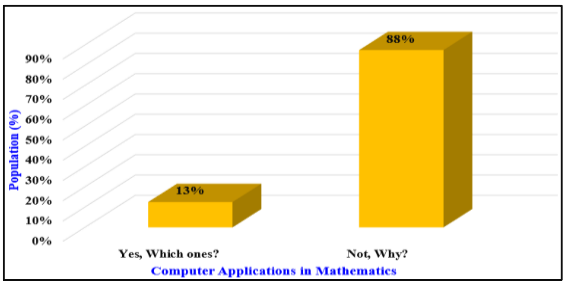
Figure 16. Analog manipulative teaching resources or playful games.



Source: Own elaboration.

Question 11: *"**Computer applications for mathematical learning**"*, describes the primary answer choice [R54]-b, based on not, with maximum value of 88% and the secondary answer choice [R53]-a, indicated with yes, for value minimum of 13%. Answer 1, indicates a lack of knowledge, lack of interest, lack of time and inability to recall. Answer 2, acknowledges using Photomath them solve problems, Excel for tabulating and graphing data, Sudoku puzzles and online card games. Conceptual decision is that all women marked option [R54]-b, according figure 17.

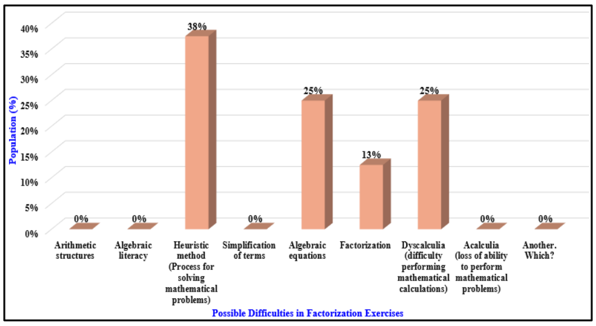
Figure 17. Computer applications in the mathematics.



Source: Own elaboration.

Question 12: *"**Difficulties for developing factorization exercises**"*, describes the main answer choice [R57]-c, based on the heuristic method (Process for solving mathematical problems) with maximum value of 38%, and the secondary answer choice [R60]-f, indicated in factor decomposition with minimum value of 13%. Conceptual decision is that 2 women marked choice [R57]-c, and 2 women marks choice [R59]-e, according figure 18.

Figure 18. Difficulties in the factorization exercises.

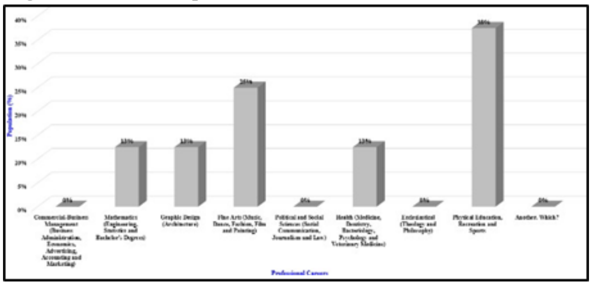


Source: Own elaboration.

Question 13: *"**Choosing a career path for the future**"*, describes the main answer option [R71]-h, based on the physical education, recreation and sports with maximum value of 38% and the secondary answer options [R65]-b, indicated in mathematics (Engineering, statistics and bachelor's degree) with 13%; [R66]-c, based on the graphic design (Architecture) with 13% and [R69]-f, structured in the health (medicine, dentistry, bacteriology, psychology and veterinary medicine) with minimum value of 13%. Conceptual decision is that 3 men marked option [R71]-h, and 1 man marks the option [R65]-b, according figure 19.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

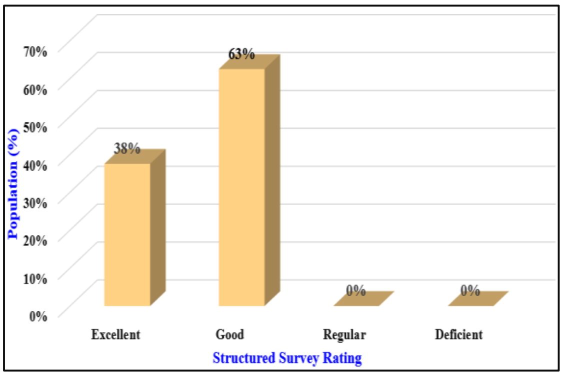
Figure 19. Careers paths for the future.



Source: Own elaboration.

Question 14: ***"The structured survey is understandable and up-to-date"***, describes the primary answer option [R74]-b, rated as good with maximum value of 63% and the secondary response option [R73]-a, rated as excellent with minimum value of 38%. Conceptual decision is that 3 men marked option [R74]-b, and 1 man marks option [R73]-a, indicating a greater preference for the male sex with first option in future professional careers, according to figure 20.

Figure 20. Understandable and up-to-date semi-open structured survey.



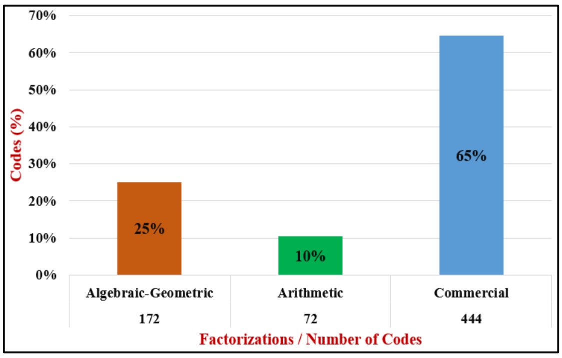
Source: Own elaboration.

Contrast between arithmetic, algebraic-geometric and commercial factorization

The maximum and minimum points of the factorization are analyzed using the column diagram that describes the percentages of codes created where the relevant of meanings characteristics are represented, using the qualitative research software program MAXQDA, according figure 21.

In ***commercial factorization***, the greatest strength of meanings is obtained through collaborative groups in the summative test with 65% and 444 codes that are in the first instance and indicate the high rates of variables based on the phases of the heuristic method, commercial units, marketing units, linear equations, quadratic equations, square roots, operationalization of calculations, utility equations, production processes, simplifications, cubic function, polynomial equation, division test of two quantities, development of ten cases of algebraic decomposition and global vision by applied problem. In ***arithmetic factorization***, the lowest strength of meanings is obtained through collaborative groups in the summative test with 10% and 72 codes that are ultimately and indicate the low rates of variables based on prime factorization, tree diagrams, use of Cuisenaire rods and numicon plates, use of the decomposition algorithm, least common multiple, operational procedures, exact successive divisions, cubic units, close the box game, unit of length, prime numbers and composite numbers.

Figure 21. The general results of codes in factorization.

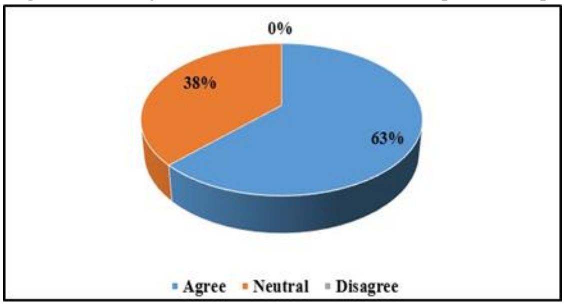


Source: Own elaboration.

Likert Attitudinal Scale "Post"

Question 1: ***"In subjects of mathematics and entrepreneurship, learn arithmetic, geometry, algebra, economics, accounting and marketing"***, describes the answer option maximum (Agree) of 63%. This important decision being preference for knowledge in the mathematics and entrepreneurship. The minimum secondary response option (Neutral) is of 38%. The conceptual decision is that 3 women marked the [Agree] option and 1 woman marks the [Neutral] option, according figure 22.

Figure 22. Subjects of mathematics and entrepreneurship.

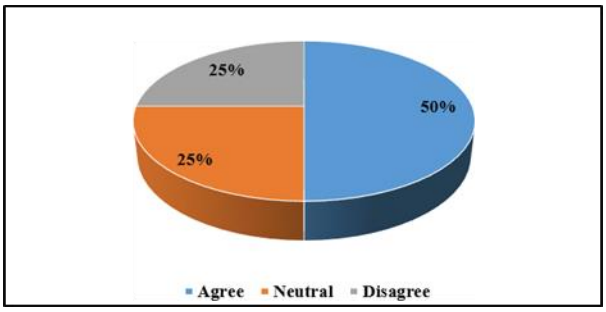


Source: Own elaboration.

Question 2: ***"Motivation for learning the ten cases of algebraic factorization"***, describes the main answer option maximum (Agree) of 50%; indicating the higher level of motivation for learning the ten algebraic factorization cases. Minimum secondary response options (Neutral and Disagree) are 25% for each one. The conceptual decision is that 2 men marked the [Agree] option and 2 men marks the [Neutral] option, according figure 23.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

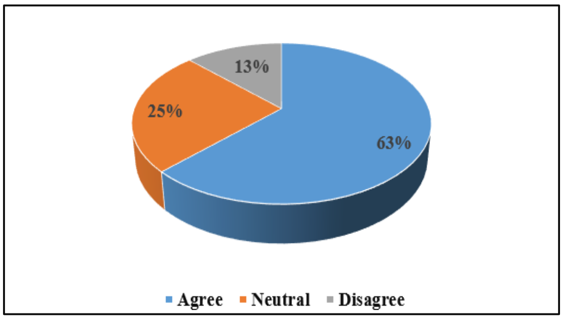
Figure 23. Learning the cases of algebraic factorization.



Source: Own elaboration.

Question 3: "Enjoys solving structural factorization processes using analytical skills", describes the answer option maximum (Agree) of 63%; indicating the higher preference for solving factorization processes using thinking skills. Minimum secondary response option (Disagree) is the 13%. Conceptual decision is that 3 men marked [Agree] option and 1 man marks [Disagree] option, according figure 24.

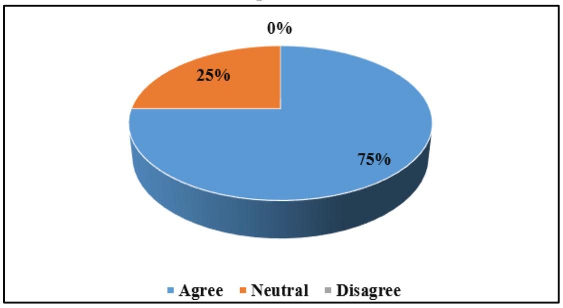
Figure 24. Resolution analytical processes in factorization.



Source: Own elaboration.

Question 4: "Business trade is recognized in proposed factorization problems", describes the answer option maximum (Agree) of 75%; indicating the higher preference for recognizing business trade in goods and services in applied algebraic factorization problems. Likewise, it is indicated that minimum secondary response option (Disagree) is the 25%. Conceptual decision is that all women marked [Agree] option, according figure 25.

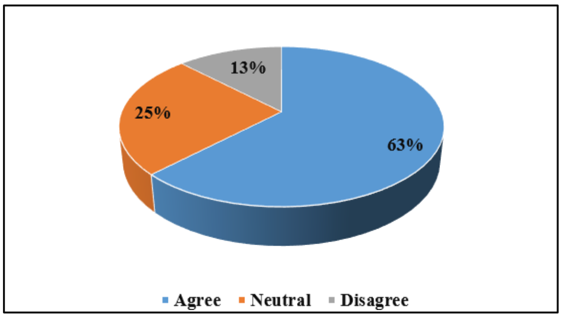
Figure 25. Application of commerce in factorization problems.



Source: Own elaboration.

Question 5: "Motivation in factorization applications, through other sciences of knowledge", describes the answer option maximum (Agree) of 63%; indicating the higher preference motivation in the application of factorization other sciences applied thinking. The minimum secondary response option (Disagree) is the 13%. Conceptual decision is that 3 men marked [Agree] option and 1 man marks [Neutral] option, according figure 26.

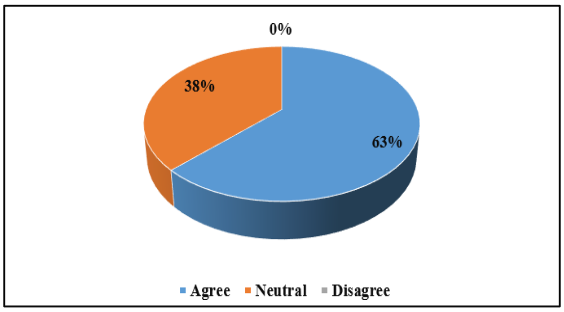
Figure 26. Applications of factorization in other sciences.



Source: Own elaboration.

Question 6: "The Motivation for learning concepts and practices in algebraic factorization through Baldor and the commercial factorization", describes the answer option maximum (Agree) of 63%; indicating the higher preference of motivation in classes to learn the notions and applications of algebraic factorization. The minimum secondary response option is (Disagree) with 38%. Conceptual decision is that 3 women marked the [Agree] option and 1 woman marks the [Neutral] option, according figure 27.

Figure 27. Applications of factorization in other sciences.

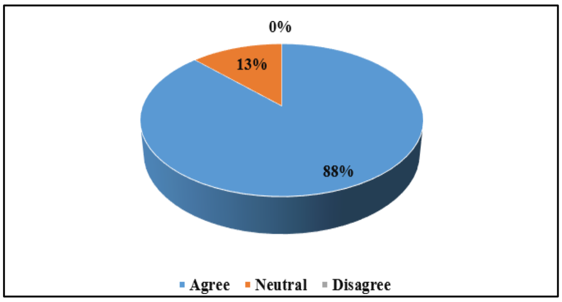


Source: Own elaboration.

Question 7: "The application problems in commercial factorization are real and up-to-date", describes the answer option as maximum (Agree) of 88%; indicating the higher preference for the knowledges in application problems in the commercial factorization, being real and up-to-date. Minimum secondary response option (Neutral) is the 13%. Conceptual decision is that all women marked [Agree] option, according figure 28.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

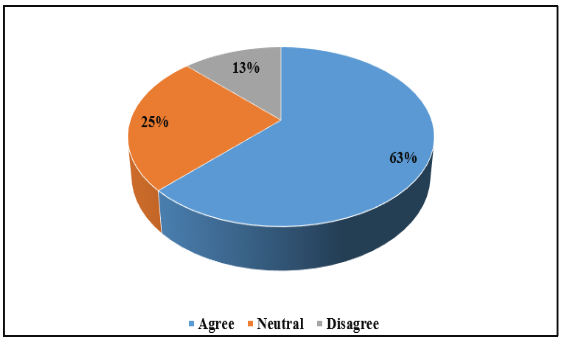
Figure 28. The application problems in commercial factorization.



Source: Own elaboration.

Question 8: ***"The motivation of images included in commercial factorization problems that generate learning of know-how",*** in where describes the answer option maximum (Agree) of 63%; indicating the higher preference motivational for the images commercial factorization problems. Minimum secondary response option (Disagree) is the 13%. Conceptual decision is that all men marked [Agree] option, according figure 29.

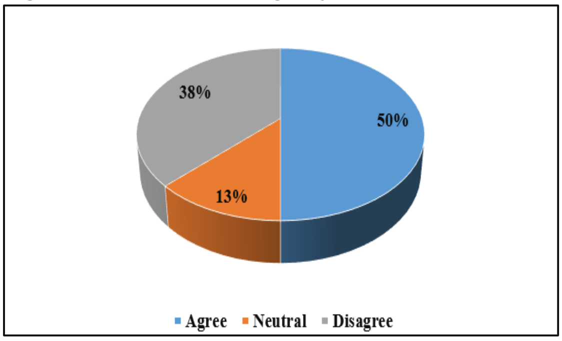
Figure 29. Images in applied problems of commercial factorization.



Source: Own elaboration.

Question 9: ***"Motivation for mathematics classes, taught by teachers with pedagogical strategies and knowledge",*** describes the answer option maximum (Agree) of 50%; indicating the higher preference motivational for the math classes taught by school's teachers. The minimum secondary response option (Neutral) is of 13%. Conceptual decision is that 3 men marked the [Agree] option and 1 man marks the [Neutral] option, according figure 30.

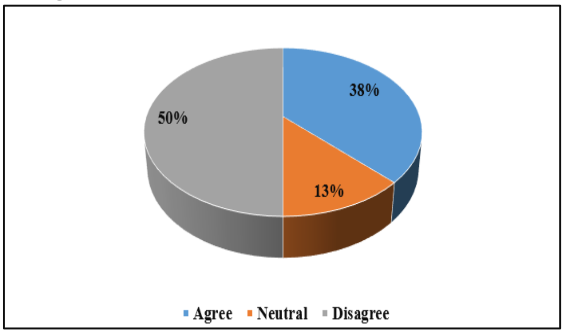
Figure 30. Math classes taught by teachers from the school.



Source: Own elaboration.

Question 10: ***"Recognition of the heuristic method in problem solving, based on the statement, approach to the question, solution and verification in commercial factorization",*** describes the main answer option maximum (Disagree) of 50%; indicating the higher level of importance where the heuristic method in solving commercial factorization problems is not recognized. Minimum secondary response option (Neutral) is the 13%. The conceptual decision is that 2 women marked the [Agree] option and 2 women marks the [Disagree] option, according figure 31.

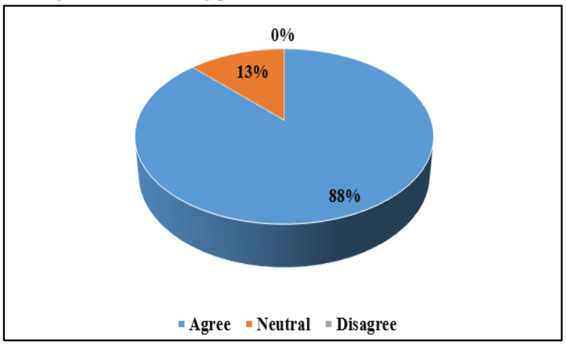
Figure 31. Heuristic method in commercial factorization.



Source: Own elaboration.

Question 11: ***"The writing statements in commercial factorization problems with motivation, being creative, understandable and practical",*** describes the main answer option maximum (Agree) of 88%; indicating the higher level of motivational importance in writing applied commercial factorization problems with innovation. The minimum secondary response option (Neutral) is of 13%. Conceptual decision is that all women marked [Agree] option, according figure 32.

Figure 32. Writing problems in commercial factorization.

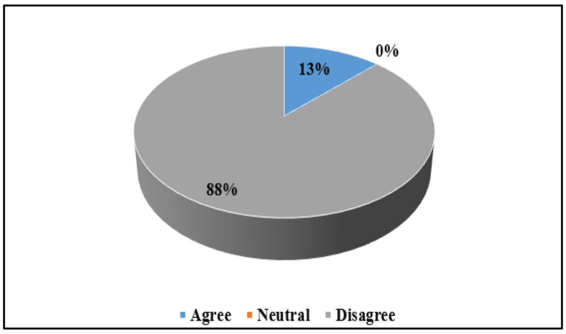


Source: Own elaboration.

Question 12: ***"Preference for remaining silent without clarifying doubts in class when the arithmetic and algebraic factorization is not understood",*** describes the main answer option maximum (Disagree) of 88%; indicating the higher preference motivational for remaining silent and avoiding questions of concepts in math class due to a lack of understanding. The minimum secondary response option (Agree) is of 13%. The conceptual decision is that all men marked [Disagree] option, according figure 33.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

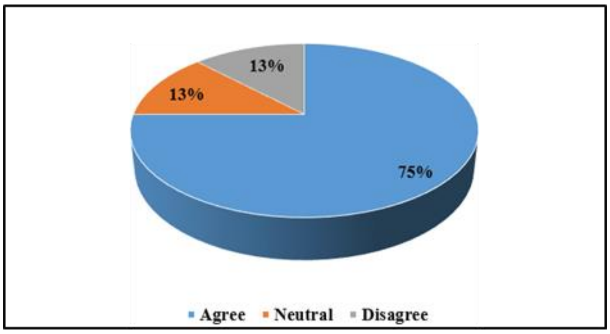
Figure 33. The learning experiences in arithmetic and algebraic factorization.



Source: Own elaboration.

***Question 13:** "**Recognition of dynamic application of Geogebra in systems classes, allowing factorization processes to be carried out with graphics of real designs**",* describes the main answer option maximum (Agree) of 75%; indicating the higher level preference in where the application of Geogebra in systems classes during the factorization with simplified operations is recognizing. Minimum secondary response options (Neutral and Disagree) are 13% each. Conceptual decision is that 3 men marked the [Agree] option and 1 man marks the [Neutral] option, according figure 34.

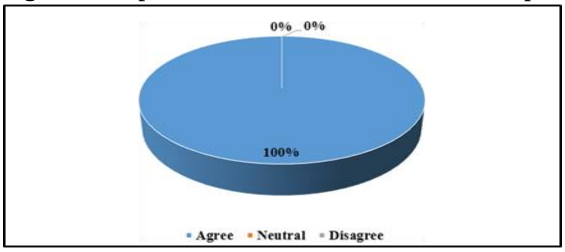
Figure 34. Application of Geogebra in systems classes.



Source: Own elaboration.

***Question 14:** "**Motivation for collaborative work in pairs that contributes to the generation of new knowledge, ideas and coexistence**",* describes the main answer option maximum (Agree) of 100%; indicating the higher level motivational importance the experience of collaborative work in pairs for the new knowledge. The minimum secondary response options (Neutral and Disagree) are 0% each. Conceptual decision is that all women and men marked [Disagree] option, according figure 35.

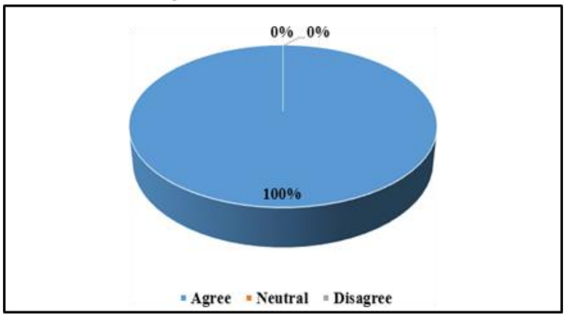
Figure 35. Experiences of the collaborative work in pairs.



Source: Own elaboration.

***Question 15:** "**Motivation for learning applied problems in commercial and geometric factorization, without the development of routine algebra exercises**",* describes the main answer option maximum (Agree) of 100%; indicating the higher level motivational component for learning commercial and geometric factorization problems applied. The minimum secondary response options (Neutral and Disagree) are 0% each. Conceptual decision is that all women and men marked the [Agree] option, according figure 36.

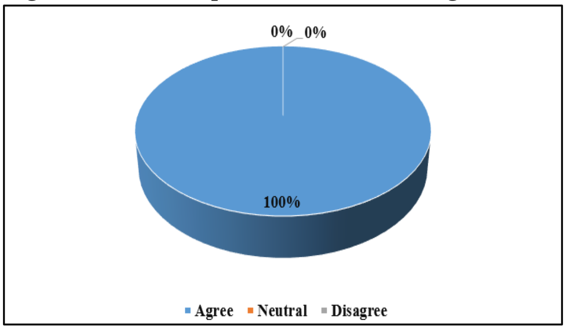
Figure 36. Learning of problems on commercial and geometric factorization.



Source: Own elaboration.

***Question 16:** "**Manipulative resources and games/contests allow for the creation and recreation of constructive factorization learning arithmetic, algebraic, geometric and commercial**",* describes the main answer option maximum (Agree) of 100%; indicating the higher level motivational component of knowledges in concrete manipulative resources and games-contests in applied and creative factorization. The minimum secondary response options (Neutral and Disagree) are 0% each. This conceptual decision describes that all women and men marked the [Agree] option, according figure 37.

Figure 37. The manipulative resources and games-contests.

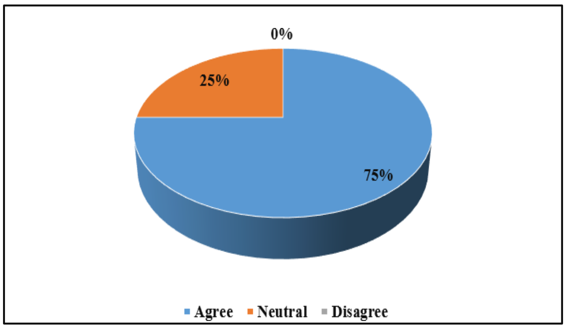


Source: Own elaboration.

***Question 17:** "**The application commercial factorization motivates the implementation of business opportunity, product or service in the future**",* describes the main answer option maximum (Agree) of 75%; indicating the higher level motivational component for to create a business, service or products in commercial factorization. The minimum secondary response option (Neutral) is of 25%. Conceptual decision is that 3 women and 3 men marked the [Agree] option. Besides, 1 woman and 1 man marks the [Neutral] option, according described in figure 38.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

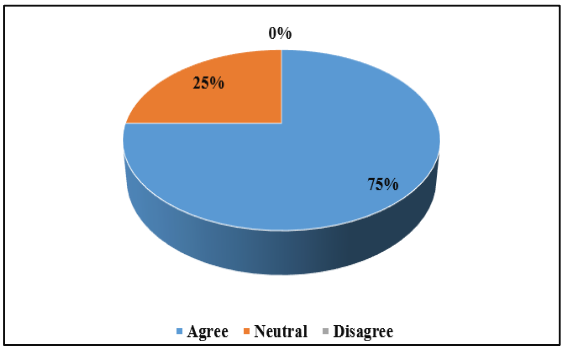
Figure 38. The application Commercial factorization to create a business or product.



Source: Own elaboration.

Question 18: ***"The motivation in social entrepreneurship classes with marketing strategies, development of the fieldwork and cognitive analysis cognitive"***, describes the main response option (Agree) of 75%; which indicates the highest level ofmotivation for the study based on applied strategies and techniques that lead to social entrepreneurship in the current businesses. Minimum secondary response option is (Neutral) is of 25%. Conceptual decision is that 3 women and 3 men marked the [Agree] option. Also, 1 woman and 1 man marks the [Neutral] option, according described in figure 39.

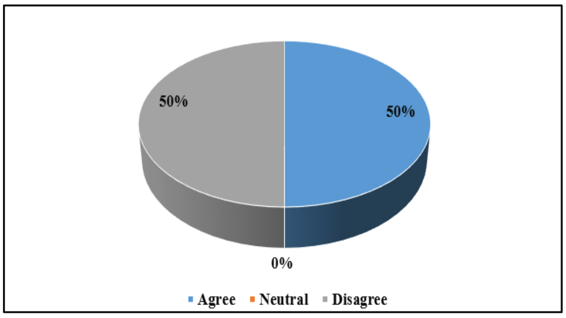
Figure 39. Social entrepreneurship classes in business.



Source: Own elaboration.

Question 19: ***"Recognition of Python-Matplotlib programming in the systems classes applied, creating logical structures, data analysis, algorithms and graphs"***, describes the mains responses options (Agree and Disagree) of 50% each; which indicating a high level of importance, recognizing the study Python-Matplotlib programming with constructive algorithm development during systems classes. The indicated minimum secondary response option is (Neutral) with 0%. Conceptual decision is that 2 women and 2 men marked the [Agree] option. Besides, 2 women and 2 men marks the [Disagree] option, according described in figure 40.

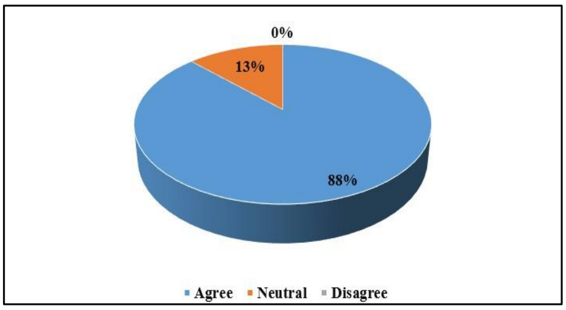
Figure 40. The application of Python-Matplotlib programming in systems classes.



Source: Own elaboration.

Question 20: ***"Motivation of the problem-based learning to apply the STEM model in science, technology, engineering and mathematics"***, describes the main answer option maximum (Agree) of 88%; which indicating a high level of motivational importance for developing problem-based learning with the STEM, model without dividing knowledge. Indicated minimum secondary response option is (Neutral) with 13%. Conceptual decision describes that all women marked the [Agree] option and 2 women marks [Disagree] option, according described in figure 41.

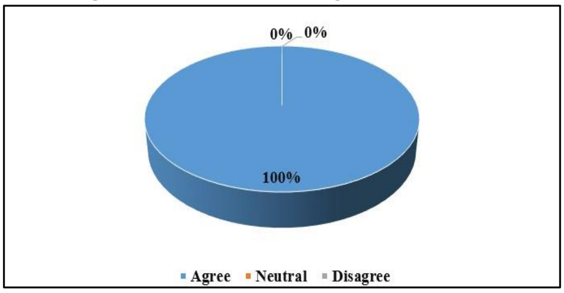
Figure 41. Problem-based learning with STEM application.



Source: Own elaboration.

Question 21: ***"Motivation for the content and design of Likert attitudinal scale with concise, objective and spontaneous responses in benefit of the mathematical didactics"***, describes the main answer option maximum (Agree) of 100%; respect to the content and design of the evaluative attitude scale of educational project. The minimum secondary response options (Neutral and Disagree) are 0% each. This conceptual decision describes that all women and men marked the [Agree] option, according described in figure 42.

Figure 42. Content and design of attitudinal scale.



Source: Own elaboration.

Likert attitude scale structure

The Likert attitudinal scale is structured according to the

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

cognitive approach, which indicates prior knowledge about the subject being learned. Additionally, the affective approach describes what motivates or attracts the student for learn. Consequently, the conative approach generates intrinsic motivation or emotional enjoyment of the subject being learned topic.

The maximum and minimum points of the frequency histogram describe the respective percentages according to Likert attitudinal structure with the frequency polygon that generates an ascending path from the cognitive attitude to the conative between its midpoints, as described in figure 43.

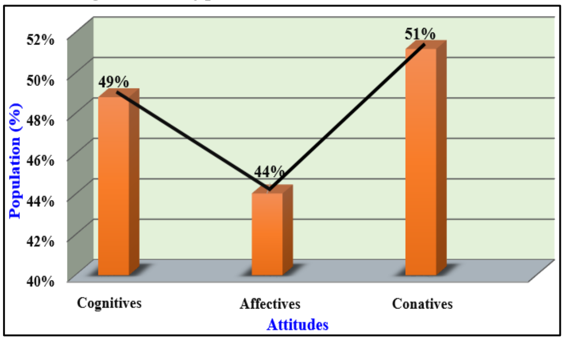
In the ***conative attitude***, maximum value of 51% is analyzed; which indicates higher level of motivational importance for the didactics activities focused on factorization of type arithmetic, algebraic, geometric and commercial in where 8 students manifest an inner psychic impulse that leads them to the action of learning.

Furthermore, emotional satisfaction has generated desire for personal improvement in the processes of structural-analytical decomposition. This involves developing classes with algebraic-commercial concepts and practices, using mathematics taught by teachers, active participation in clarifying doubts, solving commercial and geometric problems with real, applications without rote memorization exercises and developing entrepreneurship classes for the creation of strategic businesses.

In the ***affective attitude***, minimum value of 44% is analyzed; which indicates lower level of the motivational importance for the implementation didactics activities focused on factorization type arithmetic, algebraic, geometric and commercial in where the 8 students manifest a reduced attraction for action of learning.

Besides, few desires learning are described in ten cases of algebraic factorization with combined applications to other sciences, based on the images included of type commercial and the problems solving. This difficulty allows us to establish risks in the development of knowledge where the obstacles of collaborative work in pairs due to lack of coexistence is analyzed, inability to create businesses or products through commercial factoring and a minimal level of learning in applied problems with the STEM model in practical cases.

Figure 43. Types of attitudes on the Likert scale.



Source: Own elaboration.

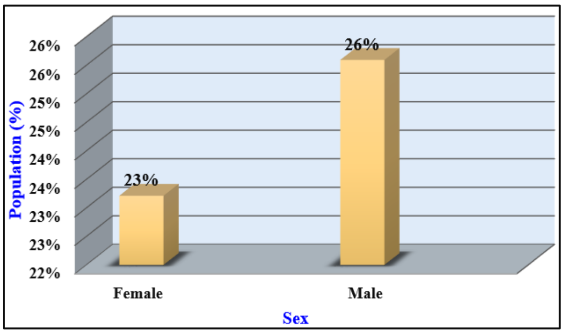
Cognitive histogram by gender, describes that the male sex represents the worth superior of 26%, where the 4 students demonstrated good attitudes of knowledge for learning in the factorization arithmetic, algebraic, geometric and commercial.

Strengths were identified in mathematics and entrepreneurship subjects, application problems in commercial factorization, heuristic method, analogous manipulative resources, games/contests, application of GeoGebra and Python-Matplotlib in the computer science classes.

Besides, the lower value corresponds to the female sex of 23%, evaluating the learning of the factorization arithmetic, algebraic, geometric and commercial with difficulties for the understanding of mathematics and business applied to computer.

Therefore, 4 students show little knowledge in learning in factorization, recognizing difficulty in the development of applied commercial problems with heuristic method and the use of Geogebra with Python-Matplotlib, as described in figure 44.

Figure 44. Cognitive attitude by gender.



Source: Own elaboration.

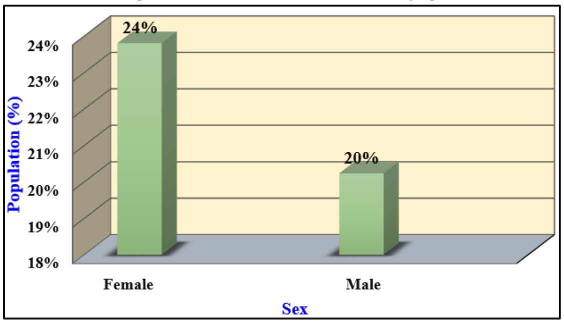
Affective histogram by gender, describes that the female sex, represents the worth superior of 24%, where the 4 students demonstrated good attitudes of motivation for learning in the factorization arithmetic, algebraic, geometric and commercial.

Strengths recognized in the understanding of ten cases of algebraic factorization, with a desire for application to other sciences, motivation by images included in business problems, understanding in the writing of statements of business factorization, a desire for work collaboratively pairs, desire to create new business opportunities or products, and a desire of problem-based learning applying the STEM model.

Also, the lower value corresponding to the male sex of 20% is evaluated, where the 4 students show little motivation in the cases of factorization with applications to other sciences, little desire for the images included in the problems, little desire in the writing of commercial statements, little desire for collaborative work, little desire to generate business or products opportunities and little desire for the problem-based learning, as described in figure 45.

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

Figure 45. Affective attitude by gender.



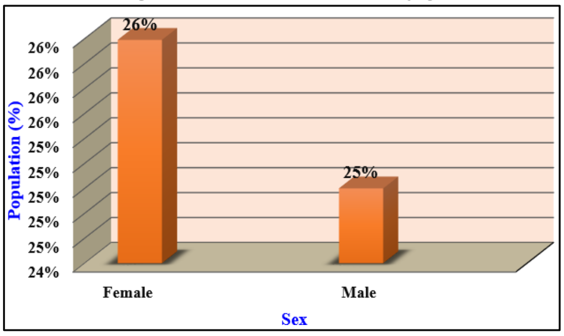
Source: Own elaboration.

Conative histogram by gender, describes that the female sex, represents the worth superior of 26%, where the 4 students demonstrated good inner impulses for learning in the factorization arithmetic, algebraic, geometric and commercial.

Strengths are recognized in the emotional desires for structural and analytical math decomposition processes, desires of classes in algebraic and commercial, desires for math classes taught by the school teachers, desires for active participation that involves clarifying doubts class, desires for the business and geometric the problems with real applications without routine exercises, desires for entrepreneurship classes for business creation, and desires for the design of content on the Likert attitude scale.

In addition, the lower value corresponding to the male sex of 25% is evaluated, where the 4 students show little desire for the processes of analytical structure in the math decomposition, little desire for the development of classes with algebraic and commercial practices, little desire for the mathematics classes taught by teachers, little desire in active participation for the clarification of doubts, little desire in commercial and geometric problems without routine exercises, little desire in entrepreneurship classes for the creation of businesses and little desire in the content-design of attitude scale, according figure 46.

Figure 46. Conative attitude by gender.



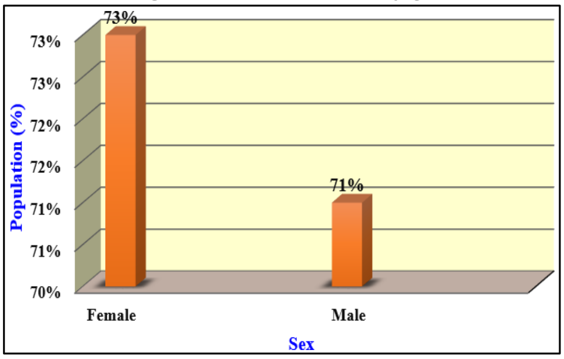
Source: Own elaboration.

Contrast between female and male sexes

The global a comparison by gender is performed, describing the highest value corresponding to the ***female sex*** at 73%. Thus, the most important quality is described among cognitive attitude (23%), affective attitude (24%) and conative attitude (26%). It is based on the higher percentage of the ***conative attitude***, based on the inner desire for motivation towards learning in the factorization arithmetic, algebraic, geometric and commercial with its pedagogical methodologies, teaching-learning strategies and the resources of material and virtual didactic application.

Also, the minimum value corresponding to the ***male sex*** is evaluated with 71%. Thus, the lesser important quality is described among cognitive attitude (26%), affective attitude (20%) and conative attitude (25%). It is based on the lower percentage of ***affective attitude*** that describes the low efficiency of attraction in the student's attitudes towards problem-based learning of the factorization arithmetic, algebraic, geometric and commercial, with little external motivation for collaborative work and the creation of micro-enterprises or products, according figure 47.

Figure 47. Total attitudes by gender.



Source: Own elaboration.

6. CONCLUSIONS

The study indicates the answer to the research question where the students of the educational center have been strengthened with collaborative and constructive learning in resolution of commercial factorization problems, applying the ten cases of algebraic decomposition with arithmetic and geometry, developing virtual and playful programs with games/contests that are based on marketing magnitudes and economic equations that induce the culture of consumption in the current companies in where globalization generates productivity.

In this way, commercial management is based on entrepreneurship, recognizing the buying and selling of products described in the development of mathematical problems that define their implementation during the profitability process. Besides, the virtual environments create and recreate algebraic-commercial learning, fostering critical thinking, cognitive development, logical analysis and the creative and exploratory implementation of experiences that lead to trial-and-error proposals, generating active and meaningful learning in a visual and interactive way.

The theoretical categories, conclude with contributions and recommendations:

Characteristics of arithmetic, algebraic, geometric and commercial factorization.

Contributions emphasize the development of critical thinking within mathematical structures, based on analytical processes of algebraic-commercial factorization. It is recommended to

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

create project-based learning with innovative designs that allow for the implementation of the STEM model, starting from being, knowing, knowing-how and learning to think.

Problem-based learning applying the collaborative work. Contributions emphasize the development of heuristic method that describes the strategic mental steps in solving algebraic-commercial cases, leading to cognitive decision-making. It is recommended to apply the resolution of structured and unstructured mathematical problems in real world scenarios to deepen understanding of their deductive combinations through constructive group work.

Application of concrete and digital manipulatives resources. Contributions emphasize the creative and dynamic development of constructive knowledge through tangible and virtual pedagogical tools, facilitating the concrete, pictorial, and abstract approaches. It is recommended to use educational games and contest to energize the learning process and socio-emotional skills in the applied mathematics, encouraging algebraic and commercial study through theory and practice with meaningful, multisensory, and active thinking.

7. LIMITATIONS/PROSPECTIVES

Limitations

During the construction of the ***theoretical framework***, difficulties arose in the search for specific information, based on the characteristics of commercial factorization, because it is an educational topic in applied mathematics it has not been investigated by the scientific community is ignored with cognitive difficulties in his development during problem solving.

The technological applications with the ***Geogebra program and Python-Matplotlib programming*** can generate problems in their handling of algorithmic structure with demotivation, cognitive confusion of the heuristic method, obstacles in visualization of the mathematical ideas, frustration, overwhelm and difficult in the exploration of virtual environment for the constructive didactic learning.

The ***delays in timing and restrictions on schedules and days*** have hindered the project's planning, due to the different school activities scheduled by educational center and the district education secretariat of the national government.

Prospectives

It would be interesting to ***continue exploring commercial factorization, applying the financial mathematics*** based on solving problems that arise as a consequence of the time value of money, from specific conditions, using concepts and equations with the qualitative research method, cooperative learning and the games/contests, developing's Cabri Express program with visual Scratch programming.

There is the ***possibility of expanding the sample of students*** in cooperative groups, with the participation of two educational centers (one private and one public) where the results between two high school courses are compared (10th and 11th) in applied mathematics, generating a constructive learning with the STEM model.

8. ACKNOWLEDGMENTS

I thank my parents for their motivational support. The teachers for their pedagogical knowledge. The supervisor of master's thesis for her research orientation. The directors and teachers of the mathematics area, in morning session of the educational center, for having accepted the research proposal for student didactic learning in educational improvement. The chosen students, for their dedication, motivation, interest and respect. And the family parents for the authorization granted through UNIR informed consent.

Sources of funding come from the author's personal for savings that were allocated to the *cover operational economic expenses* (printing and stationery), the *expenses on consumer goods* (perishable food), the *logistical economic expenses* (transport), the *expenses on educational materials* (analog manipulative teaching resources, games/contests, audiovisual resources and technological resources with application virtual programs).

REFERENCES

1. Baldor, A. (2016). *Algebra* (cap. X, pp. 143-172). Homeland. Retrieved from: https://www.yumpu.com/es/document/read/54936190/algebra-de-baldor-2da-edicion/19
2. Bermúdez, J. (2021). Problem-Based Learning to Improve Critical Thinking: Systematic Review. *INNOVA Magazine Research Journal, 6*(2), 77-89. https://dialnet.unirioja.es/servlet/articulo?codigo=8226162
3. Bernaschini, E. (2017). Prime Numbers: An EndlessStory. *Journal of Mathematical Education, 32* (3), 29-36. https://revistas.unc.edu.ar/index.php/REM/article/view/18734
4. Bolaños, M. & Segovia, I. (2021). Structural Sense of First-Year University Students. *Uniciencia Magazine, 35*(1), 152-168. https://www.scielo.sa.cr/pdf/uniciencia/v35n1/2215-3470-uniciencia-35-01-152.pdf
5. Castellano, Z. & Sandoval, A. (2022). Deterministic Factorizer Primordial Sequence Tesla Zollner Prime and Composite Numbers. *Tesla Zollner-TECSZOLL, C.A. Multidisciplinary Research Group Journal,* 1-10. https://www.researchgate.net/profile/ZollnerCastellano-2
6. Damián, I., Benites, L. & Camizán, H. (2021). Collaborative Learning as a Didactic Strategy in Latin America. *Tecnohumanismo Magazine, 1*(1), 203-225. https://doi.org/10.53673/th.v1i8.41
7. De León, I. & Ávila, Y. (2024). Geometric Algebra as a Mediator to Strengthen Factorization Processes. *Multidisciplinary Scientific Journal Latin Science, 8* (5), 4894-4912. https://ciencialatina.org/index.php/cienc

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

iala/article/view/13937/19982

8. Fernández, R., Hernández, C., Prada, R. & Ramírez, P. (2020). Beliefs and AnxietyTowards Mathematics: A Comparative Study Between Teachers from Colombia andSpain. *Bolema Magazine (Boletim de Educação Matemática), 34*(68), 1174-1205.https://www.scielo.br/j/bolema/a/hL4J4HWdZh6Q6kn9F8ZnmdG/?lang=es
9. Flores, N., Pastrana, M. & Flores, W. (2017). Assessment Strategies in Teaching Factorization Algorithms in Ninth Grade of Secondary Education. *Science and Interculturality Journal, 20*(1), 7-17. https://dialnet.unirioja.es/servlet/articulo?codigo=6462002
10. Gallego, A. (2024).*Manipulative Materials and Mathematicsin Compulsory Secondary Education* [Final Master's Project, University of Valladolid]. UVA Repository. https://uvadoc.uva.es/bitstream/handle/10324/74051/TFM-G2080.pdf?sequence=1
11. García, E. & Puyol, J. (2024). Strategies to Improve the Teaching of Factorization in the Classroom: Addressing Common Student Difficulties. *The Multidisciplinary Scientific Journal G-ner@ndo, 5*(2), 2207-2219.https://revista.gnerando.org/revista/index.php/RCMG/article/view/359/359
12. Gasco, J. (2017). Arithmetic-Algebraic Problem Solving and Learning Strategies in Mathematics. A Study in Compulsory Secondary Education (ESC). *Latin American Journal of Research in Mathematics Education, 20*(2), 167-192. https://relime.org/index.php/relime/arti cle/view/122
13. González, E. (2022). *Manipulative Mathematics in Secondary Education* [Final Master's Thesis University Alcalá of Henares]. UAH Institutional Repository. https://ebuah.uah.es/dspace/bitstream/handle/10017/53973/TFM_Oliver_Gonzalez_Ortega_2022.pdf;jsessionid=274804B1BB40847129E785D5BA9B6CFE?sequence=1
14. ICFES. (2024). Programme for International Student Assessment (PISA). National Results Report for Colombia 2022. *Ministry of National Education,* 1-68. https://www.mineducacion.gov.co/1780/articles-421217_recurso_03.pdf Jiménez, L. & Espinosa, C. (2019). The Use of Manipulative Materials to Strengthen Mathematical Thinking in Multigrade Classrooms. *Education and Science Magazine,* (23), 513-529. https://dialnet.unirioja.es/servlet/articulo?codigo=7982149
15. Loor, V. (2022). The Analysis of Consumption, Production and Market. *E-IDEA Magazine Journal of Business Sciences, 4* (14), 44-56. https://doi.org/10.53734/eidea.vol4.id1 88
16. Mendoza, J., Vega, C., Silva, B. & Boy, A. (2024). Problem-Based Learning: A Perspective from the Educational Context. *Horizons Journal of Research in Educational Sciences, 8*(35), 2400-2416. https://revistahorizontes.org/index.php/revistahorizontes/article/view/1767
17. Muñoz, M., Fernández, J. & Sánchez, F. (2019). Rational and Impulsive Purchasing Behavior of Young Students in Shopping Centers. *Academia & Business Magazine, 5*(1), 61-70. https://dialnet.unirioja.es/servlet/articulo?codigo=7085734
18. Ortiz, F. (2019). A Demonstration of the Fundamental Theorem of Algebra by Julio Garavito. *Journal of Mathematical Readings, 40*(1), 73-101. https://dialnet.unirioja.es/servlet/articulo?codigo=7013017
19. Patiño, K., Prada, R. & Hernández, C. (2021). Mathematical Problem Solving and the Factors Involved in its Teaching and Learning. *Redipe Bulletin Magazine, 10*(9), 459-471. https://revista.redipe.org/index.php/1/article/view/1453/1372
20. Pozo, A. (2020). *Manipulative Materials and Problem Solving in Algebra for Secondary Education* [Final Master's Thesis, University of Valladolid]. UVA Repository. https://uvadoc.uva.es/bitstream/handle/10324/43430/TFM-G1291.pdf?sequence=1&isAllowed=y
21. Ramos, J. & Núñez, L. (2024). STEM Approach to Developing Problem-Solving Skills and its Impact on Academic Management. *Invecom Magazine: Transdisciplinary Studies in Communication and Society,* 4(2), 1- 20. https://ve.scielo.org/pdf/ric/v4n2/2739-0063-ric-4-02-e040248.pdf
22. Rivera, D. (2020). *Teaching of Factorization Through Relationship Between Algebra and Geometry* [Final Master's Thesis, University National of Colombia]. The UNAL Insti utional Repository.https://repositorio.unal.edu.co/bitstream/handle/unal/77879/1041 326751.2020.pdf?sequence=4
23. Rodríguez, M., Pineda, D. & Castro, C. (2020). Modern Marketing Trends, A Theoretical Review. *Spaces Magazine, 41*(27), 306-322. https://www.revistaespacios.com/a20v41n27/a20v41n27p26.pdf
24. Tantalean, I. (2022). The Nature of the Enterprise. Its Teleology from the Administration, Law and Economics Sciences. *Journal of the Faculty of Law and Political Science, 52*(136), 262-287. http://www.scielo.org.co/pdf/rfdcp/v5 2n136/0120-3886-rfdcp-52-136-262.pdf
25. Trejos, O. (2019). The Learning Methodology for to Decompose Natural Numbers into Prime Factors Using Functional Programming. *I+D Journal of Research, 13*(1), 32-44. https://sievi.udi.edu.co/ojs/index.php/I

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**

D/article/view/194/223

26. Vargas, W. (2021). Problem Solving and the Development of Mathematical Thinking. *Horizons. The Journal of Research in Educational Sciences, 5*(17), 230-251. https://revistahorizontes.org/index.php/revistahorizontes/article/view/169
27. Vasco, E. (2023). *Geometric Algebra as a Didactic Tool for Teaching the Factorization of Trinomials* [Undergraduate Final Project, Catholic University of Manizales]. UCM The Institutional Repository. https://repositorio.ucm.edu.co/server/api/core/bitstreams/ddcb1026-d690-4063-8c32-04f23dfb6278/content
28. Zambrano, M., Alvarado, A., Andrade, F. & Vinces, L. (2025). Game-Based Learning as a Tool for Teaching Mathematics. *The Scientific Journal of Educational Innovation and Current Society "ALCON", 5*(1), 243-257. https://doi.org/10.62305/alcon.v5i1.40 7
29. Zhou, C. & Navarro, I. (2025). The Problem-Based Learning as a Tool for Creative Learning. *Mexican Journal of Educational Research*, *30*(104), 113-132.https://dialnet.unirioja.es/servlet/articulo?codigo=9951566

*Corresponding Author: **Javier Darío Vergara Castiblanco Mg.**