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ONTOLOGY AND EPISTEMOLOGY OF CHILDREN'S MATHEMATICS: A Review of Leslie P. Steffe'sThought

By

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Abstract

This article critically reviews Leslie P. Steffe's work Children's Mathematics and Mathematics of Children by focusing on the ontological and epistemological dimensions of children's mathematics from a constructivist perspective. The study employs a library research approach combined with philosophical analysis of Steffe's central ideas concerning the nature of mathematics, the construction of children's mathematical knowledge, and the methodology of teaching experiments. The findings indicate that Steffe conceives mathematics not as an objective entity existing independently of human beings, but rather as a product of human intellectual activity that develops through mental operations and social interaction. Within this framework, Steffe distinguishes between children's mathematics as the mathematics constructed by children and mathematics of children as the conceptual model developed by researchers to explain children's mathematical activity. Furthermore, this article demonstrates that the teaching experiment approach developed by Steffe represents a constructivist methodology integrating teaching and research in order to understand the development of children's mathematical thinking in depth. The study also reveals that Steffe's thought has significant implications for curriculum, instruction, assessment, teacher education, and contemporary mathematics education research, particularly in the development of meaningful, reflective, and student-centered learning. Nevertheless, Steffe's perspective also presents certain limitations, especially due to its predominant focus on individual construction, which leaves the social and cultural dimensions of mathematics insufficiently elaborated. Overall, Leslie P. Steffe's work provides an important contribution to the reconstruction of modern philosophy of mathematics education by positioning children's mathematics as a legitimate cognitive reality that deserves to serve as a foundation for the development of mathematics education.

Keywords: *constructivism, children's mathematics, epistemology of mathematics, ontology of mathematics, teaching experiment, Leslie P. Steffe.*

INTRODUCTION

The development of modern philosophy of mathematics education has demonstrated a significant shift in emphasis from mathematics as a formal system toward mathematics as a product of human activity. This shift has given rise to various constructivist approaches that position learners not merely as recipients of knowledge, but as active subjects who construct their own mathematical understanding (Ernest, 1991; von Glasersfeld, 1995). Within this context, the work of Leslie P. Steffe constitutes one of the most important contributions to the reconstruction of how children's mathematics is understood through the perspectives of radical constructivism and teaching experiments. Steffe's article, *Children's Mathematics and Mathematics of Children*, not only proposes models of children's mathematical development, but also embodies strong ontological and epistemological positions concerning the nature of mathematics and the nature of children's mathematical knowledge (Steffe, 2025).

Steffe's article is particularly significant because it rejects the traditional view that school mathematics represents the only legitimate form of mathematics. Instead, Steffe argues that children possess their own mathematical constructions, which develop through mental activity and social interaction. From this perspective, children's mathematics is not understood as an incomplete form of formal mathematics, but rather as a cognitive reality possessing its own structure and legitimacy

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(Piaget, 1980; Steffe, 2025). Such a position carries profound implications for learning theory, research methodology, and mathematics teaching practice.

Moreover, Steffe's article is important because it presents an internal critique of the Piagetian tradition in mathematics education. Steffe began his academic career by applying Piaget's theory of cognitive development through statistical methods, yet he later concluded that such an approach was unable to explain how children genuinely construct their mathematical operations (Steffe, 2013; Steffe&Kieren, 1994). This realization led Steffe to develop a new research approach integrating teaching and research through the methodology of the teaching experiment. This methodology subsequently became one of the most influential approaches in constructivist mathematics education research (Steffe& Thompson, 2000).

From a philosophical standpoint, the article also opens a broader discussion concerning the ontology of mathematics. Steffe adopts the constructivist conviction that mathematics is a product of human intelligence rather than an objective entity existing independently of human beings (Piaget, 1980; von Glasersfeld, 1995). Consequently, children's mathematics is understood as the outcome of mental constructions that evolve through reflective operations. This perspective differs fundamentally from mathematical realism, which regards mathematics as something discovered rather than constructed (Stolzenberg, 1984).

Based on this background, the present article aims to critically review Leslie P. Steffe's work by focusing on the ontological and epistemological dimensions of children's mathematics. This review also examines the methodology of teaching experiments, the concepts of *children's mathematics* and *mathematics of children*, as well as the relevance of Steffe's thought to contemporary mathematics education. In doing so, this study seeks to illuminate Steffe's philosophical contribution to the reconstruction of the nature of mathematics learning and modern mathematics education research.

A. Overview of Leslie P. Steffe's Article

Leslie P. Steffe's article *Children's Mathematics and Mathematics of Children* is a reflective work that explains the development of Steffe's intellectual trajectory as a mathematics education researcher, beginning from the early stages of his academic career to the formulation of a constructivist approach in mathematics education research. The article is not merely autobiographical, but also theoretical and philosophical, since Steffe employs his research experiences as a basis for reconstructing the nature of children's mathematics and the methodology of mathematics education research (Steffe, 2025).

In the opening section of the article, Steffe explains that he began his research career within the Piagetian research tradition, which emphasized the application of Piaget's theory of cognitive development to mathematics education. His early research employed statistical approaches to investigate the relationship between children's cognitive development and their ability to solve arithmetic problems (Steffe, 2025). However, Steffe later became dissatisfied with this approach because, although it produced cumulative findings, it failed to provide a profound explanation of how children genuinely construct mathematical knowledge. Consequently, he characterized such research as a form of *pseudo-science* in mathematics education (Steffe, 2013).

A fundamental transformation in Steffe's thinking was triggered by two important experiences. The first occurred when he was invited to participate in a preschool education project aimed at accelerating children's cognitive development through instruction in number and measurement. Steffe rejected the assumption that operational mathematical concepts could be directly taught to preoperational children, as suggested by Bruner's (1960) readiness hypothesis. He argued instead that the construction of number could not be reduced to the outcome of direct instruction because it involves spontaneous development and reflective abstraction, as explained by Piaget (1966a, 1966b).

The second and more decisive experience emerged when Steffe began teaching preoperational children directly in order to test these assumptions. During this process, he discovered that classification, seriation, and one-to-one correspondence—considered by Piaget to be foundational for number construction—did not automatically emerge in children's activity (Steffe, 2025). This experience led Steffe to realize that he lacked an adequate model for explaining the reality of children's mathematical thinking. In other words, Piagetian theory alone was insufficient to explain how mathematical operations are actually constructed within instructional contexts.

This realization eventually brought Steffe back to his identity as a mathematics teacher and motivated him to develop the teaching experiment as a research methodology. Together with two of his doctoral students, Steffe taught first-grade elementary school students while simultaneously conducting intensive observations of the development of children's mathematical thinking (Steffe et al., 1976). Through these experiments, Steffe discovered that children employed different counting strategies and that counting operations developed progressively through the construction of mental units. These findings became the basis for the emergence of the concept of *children's counting types* and various developmental models of number that later became hallmarks of Steffe's research.

The article also demonstrates the strong influence of Ernst von Glasersfeld's radical constructivism on Steffe's thought. Together with von Glasersfeld and John Richards, Steffe conducted conceptual analyses of videotaped teaching experiments in order to construct models of children's mental operations (von Glasersfeld, 2005). Through this process, Steffe began distinguishing between *children's mathematics* as the mathematics constructed by children and *mathematics of children* as the conceptual model developed by researchers to explain children's mathematical activity. This distinction

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became one of the principal epistemological foundations of the article.

In addition to explaining the development of children's numerical models, the article also articulates Steffe's philosophical position regarding the nature of mathematics. He rejects mathematical realism, which views mathematics as an objective entity discovered by humans. Instead, following Piaget and radical constructivism, Steffe conceives mathematics as a product of human intellectual activity (Piaget, 1980; von Glasersfeld, 1995). Consequently, children's mathematics is regarded as a legitimate form of mathematics that deserves to serve as a foundation for the development of school mathematics instruction.

Overall, the article may be understood as a synthesis of empirical experience, methodological reflection, and philosophical positioning within mathematics education. It not only explains how children learn mathematics, but also reexamines fundamental questions concerning what mathematics is, how mathematics is constructed, and how mathematics education research ought to be conducted. In this respect, the article constitutes a significant contribution to the development of the contemporary philosophy of mathematics education.

B. The Ontology of Children's Mathematics

Ontology is the branch of philosophy concerned with the nature of being or the reality of an object. Within the philosophy of mathematics education, the ontology of mathematics addresses the fundamental question of the nature of mathematics itself: whether mathematics constitutes an objective entity existing independently of human beings or whether it is a product of human intellectual construction. Leslie P. Steffe's article clearly reflects the latter ontological position, namely that mathematics is not an absolute reality existing independently of human beings, but rather a product of human cognitive construction that develops through mental activity and social interaction (Piaget, 1980; von Glasersfeld, 1995).

Steffe's ontological position is strongly influenced by Piagetian constructivism and Ernst von Glasersfeld's radical constructivism. In his article, Steffe explicitly states that mathematics is "a product of the functioning of human intelligence" (Steffe, 2025). This statement indicates that mathematics is not understood as a metaphysical object existing prior to human beings, but rather as the outcome of human mental operations. Consequently, children's mathematics is regarded as a legitimate cognitive reality because it emerges from children's own intellectual activity.

This perspective differs fundamentally from mathematical realism. In mathematical realism, mathematical objects are assumed to possess an objective existence independent of human activity. Mathematics is therefore understood as something discovered rather than constructed. Steffe criticizes this position by referring to Stolzenberg (1984), who argues that modern mathematicians generally operate under the belief that mathematics is discovered rather than created. In contrast, Steffe adopts a constructivist position in which mathematics arises from human activity in organizing experience and constructing operational relationships.

Within Steffe's ontological framework, children's mathematics occupies a significant status of existence. Children are not viewed as passive individuals who merely receive formal mathematics from external sources; rather, they are regarded as active subjects who construct their own mathematical structures through activities such as counting, grouping, ordering, and operating with mental units. Accordingly, children's mathematics is not merely an "error" or a "simplified version" of formal mathematics, but rather a form of mathematics possessing an internal structure appropriate to the child's stage of cognitive development.

This ontological conception becomes especially evident in Steffe's distinction between *children's mathematics* and *mathematics of children*. *Children's mathematics* refers to the mathematics genuinely constructed by children as part of their own mental reality, whereas *mathematics of children* refers to the conceptual model developed by researchers in order to explain that reality (Steffe, 2025). In other words, children's mathematics possesses ontological existence at the level of children's lived experiences and mental operations, while the researcher's model constitutes an epistemological representation of that reality.

According to Steffe, the ontology of children's mathematics is also developmental and dynamic in nature. This is reflected in his explanation of the development of number sequences. Children do not immediately possess formal concepts of number; rather, they construct numerical understanding progressively, beginning with *perceptual unit items*, followed by *figurative unit items*, and eventually *abstract unit items* (Steffe et al., 1983). Each stage represents a distinct form of mathematical existence corresponding to the mental operations constructed by the child. Thus, the ontology of children's mathematics is not static, but rather developmental in character.

In addition to being constructive and developmental, Steffe's ontology of children's mathematics also possesses a relational dimension. Mathematics does not emerge in a vacuum; instead, it develops through children's interactions with their social and cultural environments. Steffe emphasizes that *children's mathematics* results from maturation combined with constructions arising from interaction within children's socio-cultural contexts (Steffe, 2025). This perspective suggests that the existence of children's mathematics cannot be separated from children's lived experiences, communication, and social activities.

Furthermore, Steffe argues that children's mathematics possesses ontological legitimacy equivalent, in certain respects, to formal mathematics. He rejects the assumption that only the formal mathematics of adults deserves to be called mathematics. Instead, Steffe contends that the *mathematics of children* constitutes "a legitimate mathematics" because it is grounded in rationality and

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genuine mental operations manifested by children (Steffe, 2025). This position is particularly important because it creates space for valuing children's mathematical thinking within mathematics education.

Philosophically, Steffe's ontological position may be understood as a form of ontological constructivism in mathematics education. Mathematics is conceived not as an abstract world existing independently of human beings, but as a human construction that develops through experience and reflection. Within this framework, the essence of mathematics lies not in the existence of formal objects alone, but in the activity of constructing mathematical meaning. Consequently, understanding children's mathematics means understanding how children construct their own mathematical realities through continually developing mental operations.

C. The Epistemology of Children's Mathematics

Epistemology is the branch of philosophy concerned with the nature of knowledge, the sources of knowledge, and the ways in which knowledge is acquired and validated. In the context of Leslie P. Steffe's article, the epistemology of children's mathematics concerns how children construct mathematical knowledge through mental activity and interactive experience. In contrast to transmissionist perspectives, which regard mathematical knowledge as something transferred from teacher to student, Steffe argues that mathematical knowledge is the result of children's own active construction (Steffe&Kieren, 1994; von Glasersfeld, 1995).

Steffe's epistemological position is strongly influenced by Piaget's genetic epistemology. Piaget argued that knowledge is not passively acquired from the environment, but rather constructed through mental operations that develop progressively over time (Piaget, 1980). In his article, Steffe adopts this fundamental idea, particularly the concept of *reflective abstraction*, which explains how new mental operations are constructed from previous actions. Nevertheless, Steffe later criticized the direct application of Piagetian theory in mathematics education because, in his view, the theory was insufficient to explain how children actually construct mathematical concepts within authentic instructional settings.

Steffe's epistemological critique emerged when he observed that classification, seriation, and one-to-one correspondence—considered by Piaget as foundational to the construction of number—did not automatically arise in the activities of preoperational children (Steffe, 2025). This experience led Steffe to realize that children's mathematical knowledge could not be understood solely through general developmental structures as described by Piaget. Instead, mathematical knowledge needed to be understood through analyses of the concrete mathematical activities children perform in instructional contexts.

Based on this realization, Steffe developed a more operational constructivist epistemology through the methodology of the teaching experiment. Within this approach, mathematical knowledge is understood as the result of reorganized experience constructed through interactions among children, teachers, and learning environments (Steffe& Thompson, 2000). Mathematical knowledge does not emerge instantaneously; rather, it develops through gradual processes of construction involving action, reflection, and the coordination of mental operations.

One of the most important epistemological concepts in Steffe's work is the notion of *unitizing operation*. This operation refers to children's capacity to construct mental units that subsequently function within counting activities and mathematical thinking (von Glasersfeld& Richards, 1983). According to Steffe, the development of children's mathematical knowledge can be understood through the ways children construct and coordinate these units. Consequently, number concepts are not interpreted merely as numerical symbols, but as outcomes of complex mental operations.

The epistemology of children's mathematics is also reflected in Steffe's concept of *number sequences*. Children do not immediately understand number as an abstract formal structure; rather, they construct numerical understanding progressively through counting activities. Steffe explains this development through stages such as the *initial number sequence*, the *tacitly nested number sequence*, and the *explicitly nested number sequence*. Each stage represents a progressively more sophisticated reorganization of mathematical knowledge. At the earlier stages, children still depend heavily on concrete objects or figurative representations. However, at later stages, they become increasingly capable of performing mental operations on numerical units abstractly and reversibly.

Another significant epistemological concept is Steffe's distinction between *first-order models* and *second-order models*. *First-order models* refer to the mathematical knowledge possessed by children themselves, whereas *second-order models* refer to researchers' constructions concerning children's mathematical knowledge (Steffe et al., 1983). This distinction reveals Steffe's epistemological awareness that researchers can never directly access children's mental reality. Researchers can only construct interpretive models based on observations of children's actions and utterances.

This epistemological awareness reflects the strong influence of Ernst von Glasersfeld's radical constructivism. Within radical constructivism, knowledge is not viewed as an objective representation of reality, but rather as an individual construction that is *viable* or functional within experience (von Glasersfeld, 1995). Accordingly, mathematical truth in the context of children's learning is not measured solely by its conformity to formal adult mathematics, but also by the coherence of the mental operations constructed by the child.

Furthermore, Steffe's epistemology positions teachers and researchers as integral participants in the process of knowledge construction. Within the teaching experiment, the teacher is not regarded as a neutral transmitter of knowledge, but as a participant who actively constructs learning situations

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and interpretations of children's activity (Steffe& Cobb, 1983). Consequently, the knowledge produced in mathematics education research is itself constructive and reflective in character.

Overall, Steffe's epistemology of children's mathematics rejects objectivist and transmissionist perspectives in mathematics education. Mathematical knowledge is understood as the result of active construction developing through mental operations, experience, and social interaction. From this perspective, learning mathematics is not a process of receiving preexisting knowledge, but rather a continuous process of constructing and reorganizing mathematical structures of thought.

D. Teaching Experiment as a Constructivist Methodology

One of Leslie P. Steffe's most significant contributions to mathematics education is the development of the *teaching experiment* as a constructivist research methodology. This methodology emerged from Steffe's critique of mathematics education research that relied excessively on statistical approaches and traditional experimental methods. According to Steffe, such approaches were incapable of explaining how children genuinely construct mathematical knowledge because they focused primarily on learning outcomes rather than on the underlying processes of mental construction (Steffe, 2013). Consequently, Steffe argued for the necessity of a research approach that would enable researchers to engage directly in children's learning activities.

Historically, the teaching experiment emerged when Steffe began teaching elementary school children directly while simultaneously observing the development of their mathematical thinking. Together with his doctoral students, Steffe taught first-grade students over an entire academic year and recorded their learning activities for in-depth analysis (Steffe et al., 1976). Within this process, teaching was not regarded merely as a pedagogical intervention, but rather as a scientific method for understanding children's mathematical constructions. Thus, teaching activity and research activity became fully integrated.

Unlike classical experiments, which separate researchers from research subjects, the teaching experiment positions the researcher as an integral part of the research situation. Steffe emphasized that he could not understand children's mathematics merely by observing other teachers because his own actions and interactions as a teacher significantly influenced what he observed in children (Steffe, 2025). This position suggests that research knowledge is understood as the result of interactive constructions between researchers and participants.

This perspective was strongly influenced by second-order cybernetics and radical constructivism. Within this framework, observers are never entirely neutral because they always constitute part of the world they observe (Steier, 1995). Consequently, the purpose of the teaching experiment is not to discover universal objective laws, but rather to construct conceptual models explaining how children build mathematical knowledge within specific contexts.

Methodologically, the teaching experiment possesses several defining characteristics. First, the research is conducted over an extended period of time, enabling researchers to observe the development of children's thinking continuously (Steffe& Thompson, 2000). Second, the methodology involves intensive interaction between teacher-researchers and children. Third, learning processes are documented through video recordings for retrospective analysis. Fourth, the principal objective is not to measure learning outcomes, but to construct explanations concerning children's mental operations and knowledge constructions.

Retrospective analysis constitutes a central component of this methodology. After instructional sessions are completed, researchers repeatedly review video recordings in order to identify patterns of children's thinking and mental operations (Steffe, 2025). The analyses are conducted iteratively using particular theoretical concepts to construct explanatory models of children's mathematical activity. In this context, researchers analyze not only children's observable behaviors, but also their own experiences and interpretations during the instructional process.

The concept of *conceptual analysis* also plays a crucial role within the teaching experiment methodology. Together with Ernst von Glasersfeld and John Richards, Steffe employed conceptual analysis to infer the mental operations underlying children's actions (von Glasersfeld, 1995). In this analysis, researchers ask what mental operations children must have performed in order to understand a situation in a particular way. Consequently, the focus of research shifts away from directly observable behavior toward the underlying structures of mental operations presumed to support that behavior.

Through the teaching experiment methodology, Steffe successfully developed numerous developmental models of children's mathematics, including *children's counting types*, the *initial number sequence*, the *tacitly nested number sequence*, and the *explicitly nested number sequence* (Steffe& Cobb, 1988). These models emerged not from statistical measurement, but from in-depth interpretations of children's mathematical activities during instructional processes.

In addition to its methodological contributions, the teaching experiment also carries important philosophical implications. This methodology rejects the dualism between theory and practice because theoretical understanding is constructed through instructional practice itself. Within this framework, teachers are not merely implementers of theory, but producers of scientific knowledge concerning mathematics learning. Thus, the teaching experiment expands the meaning of mathematics education research from the evaluation of learning outcomes to the reconstruction of children's mathematical thinking.

Despite its considerable influence, the teaching experiment methodology also possesses certain limitations. This form of research requires extended periods of time, limited numbers

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of participants, and highly developed reflective and theoretical capacities on the part of researchers. Furthermore, because the methodology depends heavily on researchers' interpretations, its validity is often questioned by positivist research traditions. Nevertheless, within constructivist paradigms, subjectivity is not regarded as a weakness, but rather as an inherent component of the construction of scientific knowledge (von Glasersfeld, 1995).

Overall, the teaching experiment represents a highly significant methodological contribution to modern mathematics education. Through this approach, Steffe demonstrated that understanding children's mathematics requires direct engagement in instructional processes and profound reflection upon the interactive experiences between teachers and children. Consequently, the teaching experiment is not merely a research method, but also a representation of constructivist epistemology in mathematics education.

E. Critique and Relevance of Steffe's Thought

Leslie P. Steffe's thought has made a substantial contribution to the development of constructivist mathematics education, particularly in understanding how children construct mathematical knowledge. Nevertheless, like other philosophical theories and approaches, Steffe's ideas are also open to critical examination. Critiquing Steffe does not necessarily imply rejecting his ideas; rather, it involves examining the limitations, underlying assumptions, and contemporary relevance of his perspective within mathematics education.

One of the principal strengths of Steffe's thought lies in his success in reconstructing children's mathematics as a legitimate object of scientific inquiry. Prior to the development of constructivist approaches, mathematics education tended to position formal mathematics as the primary standard, while children's errors or modes of thinking were regarded as deviations that needed correction. Steffe rejected this perspective by demonstrating that children's mathematics possesses operational structures that are rational within the context of children's cognitive development (Steffe, 2025). This perspective is highly relevant to contemporary educational paradigms that emphasize the importance of valuing students' thinking processes in mathematics learning.

Moreover, Steffe's contribution is particularly important in the development of process-oriented mathematics education research. The teaching experiment approach enables researchers to understand the development of children's mathematical knowledge in depth through direct interaction within instructional contexts (Steffe& Thompson, 2000). In contemporary research, this approach has become foundational for various forms of *design-based research* and learning research that place students' thinking processes at the center of investigation (Cobb et al., 2003).

Steffe's thought is also highly relevant to developments in mathematics education oriented toward mathematical literacy. Within modern conceptions of mathematical literacy, students' competencies are not measured solely by procedural mastery, but also by their ability to construct mathematical meaning, reason mathematically, and use mathematics flexibly across diverse contexts (OECD, 2023). Steffe's emphasis on the construction of mathematical meaning through mental operations strongly supports the development of such mathematical literacy.

Despite these significant contributions, Steffe's perspective also presents several limitations. The first criticism concerns the dominance of individual constructivism within his theory. Steffe's primary focus lies in how individuals construct mathematical structures through mental operations. As a consequence, social, cultural, linguistic, and power-related dimensions of mathematics learning receive comparatively limited attention. However, contemporary developments in the philosophy of mathematics education increasingly emphasize that mathematics is also a social and cultural practice (Ernest, 1991; Skovsmose, 1994).

From the perspective of social constructivism, mathematical knowledge is not constructed solely at the individual level, but is also shaped through communication, discourse, and negotiation of meaning within learning communities. In this regard, Steffe's thought may be considered excessively centered on individual cognitive structures. Although Steffe acknowledges the importance of social and cultural environments, his analytical focus remains primarily directed toward the individual child's mental operations (Steffe, 2025).

A second criticism concerns the complexity of the teaching experiment methodology. This approach requires extended research duration, small participant groups, intensive observation, detailed video analysis, and highly developed theoretical and reflective capacities on the part of researchers. In the context of large-scale formal education systems, such conditions are difficult to implement broadly. Teachers in schools typically face constraints related to time, large class sizes, and rigid curricular demands. Consequently, although Steffe's approach is theoretically rich, its implementation within mass education systems often encounters practical difficulties.

A third criticism relates to issues of validity and interpretive subjectivity. Within teaching experiments, researchers simultaneously function as teachers and interpreters of data. This dual role raises questions concerning the extent to which researchers' interpretations genuinely represent children's ways of thinking rather than the researchers' own conceptual projections. Positivist research traditions frequently criticize this approach as excessively subjective because it relies heavily on researchers' inferences concerning children's mental operations (von Glasersfeld, 1995). Nevertheless, within constructivist paradigms, subjectivity is understood not as a methodological weakness, but as an inherent aspect of knowledge construction itself.

Beyond methodological critiques, Steffe's thought can also be examined philosophically. His constructivist ontology of mathematics stands in opposition to mathematical realism,

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which remains influential within the broader mathematical community. According to mathematical realism, mathematical objects possess an existence independent of human thought. By contrast, Steffe conceives mathematics as a product of human intellectual activity (Steffe, 2025). This debate illustrates that Steffe's work is not merely a theory of learning, but also part of a broader philosophical discourse concerning the nature of mathematics itself.

Despite these limitations, the relevance of Steffe's thought remains remarkably strong within twenty-first-century mathematics education. As modern education increasingly emphasizes meaningful learning, differentiated instruction, conceptual understanding, and appreciation of students' ways of thinking, Steffe's ideas concerning children's mathematics become increasingly significant. Even within the context of artificial intelligence and digital learning, Steffe's perspective remains relevant because it reminds educators that the essence of mathematics learning lies not merely in obtaining correct answers, but in constructing meaningful mathematical structures of thought.

Overall, Leslie P. Steffe's thought may be regarded as one of the most significant contributions to modern constructivist mathematics education. Although his theories are not without criticism, Steffe's approach successfully broadened understandings of children's mathematics, mathematics education research, and the nature of mathematics learning itself. Consequently, Steffe's thought continues to occupy an important position in the ongoing development of the contemporary philosophy of mathematics education.

F. Implications for Mathematics Education

Leslie P. Steffe's ideas concerning *children's mathematics* and *mathematics of children* carry profound implications for mathematics education, particularly in relation to curriculum, instruction, assessment, and research. Through his constructivist perspective, Steffe reconstructs the position of students in mathematics learning from passive recipients of knowledge into active subjects who construct their own mathematical structures (Steffe, 2025). This paradigmatic shift has fundamental consequences for the ways mathematics is taught and understood in schools.

The first implication concerns the mathematics curriculum. Within traditional paradigms, curricula are often organized according to the logical structure of formal mathematics as determined by the discipline itself. As a consequence, mathematical content is frequently presented in abstract and hierarchical forms without adequate consideration of how students construct mathematical meaning. Steffe rejects this approach and argues that school mathematics should instead be grounded in *children's mathematics* (Steffe, 2025). Accordingly, mathematics curricula should be designed with careful attention to the development of students' mental operations and conceptual constructions.

This perspective is highly relevant to contemporary curricular approaches that emphasize meaningful learning and students' conceptual development. In the context of Indonesia's *Kurikulum Merdeka*, for example, mathematics learning is directed not merely toward procedural memorization, but toward conceptual understanding developed through exploration, reasoning, and reflection. Steffe's approach strongly supports such orientations because it positions conceptual understanding as the central objective of mathematics learning.

The second implication concerns the role of teachers in mathematics instruction. From Steffe's perspective, teachers are not merely transmitters of content or providers of procedural solutions, but facilitators who assist students in constructing their own mathematical structures of thought. Teachers therefore need to understand how students think, how they construct mental units, and how they develop meanings related to numbers and mathematical operations (Steffe& Cobb, 1988). Consequently, teachers' ability to interpret students' thinking becomes critically important.

This perspective implies that mathematics instruction can no longer be dominated solely by lectures and mechanical exercises. Instead, learning environments must be designed as interactive spaces that allow students to explore ideas, construct strategies, reflect upon their reasoning, and discuss their mathematical thinking. Within this framework, students' errors are not viewed merely as failures, but as important indicators of the mathematical structures currently being developed by learners.

The third implication concerns assessment in mathematics education. If mathematics is understood as a mental construction, then assessment cannot be limited to determining whether answers are correct or incorrect. Assessment must also reveal students' ways of thinking, the strategies they employ, and the mental operations underlying their problem-solving processes. Steffe's approach therefore supports the use of diagnostic assessment, clinical interviews, and analyses of students' thinking processes as essential components of mathematics evaluation.

In contemporary educational contexts, this implication aligns closely with developments in assessments emphasizing mathematical literacy and higher-order thinking skills (HOTS). OECD (2023) emphasizes that mathematical literacy involves the ability to formulate, employ, and interpret mathematics across diverse contexts. Steffe's perspective helps explain that such competencies develop through the reorganization of mathematical thinking structures rather than through procedural drill alone.

The fourth implication concerns mathematics teacher education. Steffe's thought demonstrates that mathematics teachers require deep understanding of students' cognitive development and mathematical constructions. Teachers must not only master formal mathematics, but also understand how students progressively construct mathematical concepts. Consequently, mathematics teacher education programs need to strengthen pedagogical knowledge related to analyzing students' thinking, conducting clinical interviews, and reflecting critically on mathematics learning processes.

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In addition, Steffe's approach also carries important implications for equity in mathematics education. Steffe argues that equitable mathematics instruction does not mean providing identical curricula for all students, but rather providing instruction appropriate to students' differing mathematical constructions (Steffe, 2025). This perspective is highly significant within differentiated instruction because it recognizes that each student possesses distinct mathematical developmental trajectories.

Within inclusive education and differentiated learning contexts, Steffe's ideas help teachers understand that differences in students' abilities are not merely differences in intelligence, but also differences in conceptual structures and mathematical experiences. Consequently, mathematics instruction must become more flexible and responsive to students' diverse learning needs.

Another equally important implication concerns mathematics education research. The teaching experiment methodology developed by Steffe demonstrates that mathematics education research cannot rely solely on quantitative measurements of learning outcomes. Research must also investigate how mathematical knowledge is constructed through instructional interaction (Steffe& Thompson, 2000). This approach subsequently influenced the development of various forms of design-based research and studies focusing on students' conceptual development.

In the era of digital technologies and artificial intelligence, Steffe's thought remains highly relevant. As technology becomes increasingly capable of providing immediate solutions to mathematical problems, the challenge of mathematics education is no longer merely enabling students to obtain correct answers, but helping them construct meaningful mathematical understanding. In this regard, Steffe's perspective reminds educators that the essence of mathematics learning lies in mental construction and the development of students' mathematical ways of thinking.

Overall, the implications of Leslie P. Steffe's thought for mathematics education are both broad and profound. His ideas not only transform understandings of students within mathematics learning, but also reconstruct the roles of teachers, curricula, assessment, and mathematics education research itself. Consequently, Steffe's contributions remain highly relevant in efforts to develop mathematics education that is more humanistic, reflective, and meaningful.

G. Conclusion

Leslie P. Steffe's article *Children's Mathematics and Mathematics of Children* constitutes an important contribution to the development of the philosophy of mathematics education and constructivist mathematics education research. Through this work, Steffe not only reflects upon his intellectual journey as a mathematics education researcher, but also reconstructs the ways in which the nature of children's mathematics, mathematics learning, and mathematics education research methodology are understood (Steffe, 2025).

From an ontological perspective, Steffe conceives mathematics as a product of human intellectual activity rather than as an objective entity existing independently of human beings. Within this framework, children's mathematics is understood as a legitimate cognitive reality because it emerges from children's own mental operations and experiences (Piaget, 1980; von Glasersfeld, 1995). Consequently, children's mathematics is not regarded as an inferior or incomplete form of mathematics, but rather as a form of mathematics possessing structures and rationality appropriate to children's developmental stages.

From an epistemological perspective, Steffe rejects transmissionist views that regard mathematical knowledge as something directly transferred from teachers to students. Instead, mathematical knowledge is understood as the outcome of active construction through mental activity, reflection, and social interaction. The concepts of *children's mathematics* and *mathematics of children* demonstrate the distinction between the mathematics genuinely constructed by children and the conceptual models developed by researchers to explain those constructions (Steffe, 2025). This distinction reflects the depth of Steffe's epistemological awareness within mathematics education research.

Beyond his philosophical contributions, Steffe also made significant methodological contributions through the development of the teaching experiment. This methodology enables researchers to understand the development of children's mathematical thinking through direct interaction within instructional processes. Within this approach, teaching activity and research activity become integrated, such that mathematics education research focuses not only on learning outcomes, but also on the processes through which students construct mathematical knowledge (Steffe& Thompson, 2000).

Nevertheless, Steffe's thought also presents certain limitations. His strong emphasis on individual construction results in relatively limited attention to the social, cultural, and political dimensions of mathematics education, as emphasized within social constructivism and critical mathematics education (Ernest, 1991; Skovsmose, 1994). Furthermore, the complexity and intensive nature of the teaching experiment methodology make it difficult to implement widely within large-scale educational systems.

Despite these limitations, Steffe's ideas remain highly relevant to contemporary mathematics education. As modern mathematics education increasingly emphasizes meaningful learning, differentiated instruction, mathematical literacy, and appreciation of students' ways of thinking, Steffe's conception of children's mathematics becomes increasingly important. Even in the era of digital technology and artificial intelligence, Steffe's thought reminds educators that the essence of learning mathematics lies in the construction of meaning and the development of students' mathematical structures of thought.

Overall, Leslie P. Steffe's article demonstrates that mathematics education cannot be separated from

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philosophical questions concerning the nature of mathematics and the nature of knowledge itself. Through his constructivist approach, Steffe successfully broadened understandings of children's mathematics while simultaneously reconstructing the ways mathematics education research is conducted. Therefore, Steffe's thought deserves to be regarded as one of the fundamental pillars in the development of the modern philosophy of mathematics education.

REFERENCES

1. Bruner, J. S. (1960). *The process of education*.Vintage Books.https://archive.org/details/processofeducati0000brun
2. Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. https://doi.org/10.3102/0013189X032001009
3. Ernest, P. (1991). *The philosophy of mathematics education*.Falmer Press. https://doi.org/10.4324/9780203058923
4. OECD. (2023). *PISA 2022 assessment and analytical framework: Mathematics, reading, science, financial literacy and creative thinking*. OECD Publishing.https://www.oecd.org/content/dam/oecd/en/publications/reports/2023/08/pisa-2022-assessment-and-analytical-framework_a124aec8/dfe0bf9c-en.pdf
5. Piaget, J. (1966a). General psychological problems of logico-mathematical thought. In E. W. Beth & J. Piaget (Eds.), *Mathematical epistemology and psychology* (pp. 163–190). D. Reidel Publishing Company. https://archive.org/details/mathematicalepis00beth
6. Piaget, J. (1966b). Some convergences between formal and genetic analyses. In E. W. Beth & J. Piaget (Eds.), *Mathematical epistemology and psychology* (pp. 259–280). D. Reidel Publishing Company. https://archive.org/details/mathematicalepis00beth
7. Piaget, J. (1980). The psychogenesis of knowledge and its epistemological significance. In M. Piattelli-Palmarini (Ed.), *Language and learning: The debate between Jean Piaget and Noam Chomsky* (pp. 23–34).Harvard University Press. https://philpapers.org/rec/PIATPO-3
8. Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Kluwer Academic Publishers.https://www.amazon.com/Towards-Philosophy-Critical-Mathematics-Education-ebook/dp/B00HWVWV8E
9. Steffe, L. P. (2013). Establishing mathematics education as an academic field: A constructivist odyssey.*Journal for Research in Mathematics Education, 44*(2), 353–370. https://doi.org/10.5951/jresematheduc.44.2.0353
10. Steffe, L. P. (2025). Children's mathematics and mathematics of children. In X. Yan et al. (Eds.), *Where is the mathematics in your math education research?* (pp. 249–267). Springer.https://doi.org/10.1007/978-3-031-83907-8_16
11. Steffe, L. P., & Cobb, P. (1983).The constructivist researcher as teacher and model builder.*Journal for Research in Mathematics Education*, *14*(2), 83–94. https://doi.org/10.2307/748576
12. Steffe, L. P., & Cobb, P. (1988).*Construction of arithmetical meanings and strategies*.Springer.https://doi.org/10.1007/978-1-4612-3844-7
13. Steffe, L. P., &Kieren, T. (1994).Radical constructivism and mathematics education.*Journal for Research in Mathematics Education, 25*(6), 711–733.https://doi.org/10.2307/749582
14. Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. Kelly & R. Lesh (Eds.), *Research design in mathematics and science education* (pp. 267–307).Lawrence Erlbaum Associates.https://doi.org/10.4324/9781410602725.CH11
15. Steffe, L. P., Hirstein, J., & Spikes, C. (1976).*Quantitative comparison and class inclusion as readiness variables for learning first grade arithmetic content* (Technical Report No. 9). Project for Mathematical Development of Children.https://files.eric.ed.gov/fulltext/ED144808.pdf
16. Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983).*Children's counting types: Philosophy, theory, and application*. Praeger. https://www.amazon.com.au/Childrens-Counting-Types-Philosophy-Application/dp/0275910849
17. Steier, F. (1995). From universing to conversing: An ecological constructionist approach to learning and multiple description. In L. P. Steffe& J.
18. Gale (Eds.), *Constructivism in education* (pp. 67–84). Lawrence Erlbaum Associates.https://doi.org/10.4324/9780203052600
19. Stolzenberg, G. (1984). Can an inquiry into the foundations of mathematics tell us anything interesting about mind? In P. Watzlawick (Ed.), *The invented reality* (pp. 257–308). W. W. Norton & Company. https://books.google.co.id/books/about/The_Invented_Reality.html?id=dvkVKAAACAAJ&redir_esc=y
20. Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. Falmer Press. https://files.eric.ed.gov/fulltext/ED381352.pdf
21. Von Glasersfeld, E. (2005). Thirty years constructivism.*Constructivist Foundations, 1*(1), 9–12. http://constructivist.info/1/1/009
22. Von Glasersfeld, E., & Richards, J. (1983). The creation of units as a prerequisite for number. In L.

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P. Steffe, E. von Glasersfeld, J. Richards, & P. Cobb (Eds.), *Children's counting types: Philosophy, theory, and application* (pp. 1–20). Praeger.https://www.amazon.com.au/Childrens-Counting-Types-Philosophy-Application/dp/0275910849

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